Micro and Macro Mechanical Assessment of Small and Intermediate Strain Properties of Granular Material

Dissertation

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This work is dedicated to my beloved parents
Abstract

The goal of this research is to explore the influence of fines content and stress induced anisotropy on the small and intermediate strain properties ($G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$) of granular materials from microscopic to macroscopic level. Accordingly, the work is divided into two parts: in the first part the influence of fines content is examined and in the second part the influence of stress-induced anisotropy is assessed.

(1) The resonant column tests were conducted on clean Hostun Sand to detect the influence of mean effective stress, $p'$, and void ratio, $e$, on $G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$. Then, the effect of fines on the maximum shear modulus, $G_{\text{max}}$, $G(\gamma)$ and damping ratio, $\eta(\gamma)$, was investigated with a systematic increase in fines content, $f_c$, up to 40%. The experimental results revealed that $G_{\text{max}}$ decreased with an increase in $e$ and $f_c$. Furthermore, the experimental results showed that damping ratio increased with an increase in $f_c$ up to $f_c$ of 20% and it then decreased with further increase in fines content. Micro CT scans demonstrated that sand with fines develops two different micro-structures: “fines-in-sand” and “sand-in-fines”. For “fines-in-sand”, fine particles are partially active in the sand force structure and for “sand-in-fines”, sand particles float in fine particles. The void ratio, $e$, does not capture the force structure or fabric of the sample and only represents the density of the sample. The equivalent granular void ratio, $e^*$, is a parameter for the density and fabric of the sample in transition soils. Analyses conducted in this study revealed that $e^*$ in comparison with $e$ provides a unique relationship between $G_{\text{max}}$, $f(p')$ and $f(e^*)$ in space, where the fitting parameters of Hardin’s relationship are the same as clean sand. This unique relationship can be used to predict the small and intermediate strain properties of granular materials containing fines based on test results for the clean host material.

(2) The resonant column device was modified for applying the additional vertical stress inside the samples for the anisotropic stress state tests. Simulation using FEM and piezo-electric elements were employed for the calibration and validation of the modified device. Then a series of stress induced anisotropic tests were conducted on Hostun Sand specimens for various stress paths (sp): isotropic loading, sp I ($\sigma_3 = \text{constant}$), sp II ($p' = \text{constant}$), sp III ($K = \text{constant}$), where $\sigma_1$ and $\sigma_3$ are the principal stresses, $p' = (\sigma_1 + 2\sigma_3)/3$ and $K = \sigma_3/\sigma_1$. Experimental results revealed that the impact of stress induced anisotropy on the small and intermediate strain properties of dry Hostun Sand depends on the adopted stress path. In sp I, the effect of the stress ratio on $G_{\text{max}}$ was significant but the influ-
ence of the stress ratio on $G(\gamma)/G_{max}$ and $\eta(\gamma)$ was not significant. The experimental results reveal a significant influence of stress induced anisotropy on $G_{max}$, $G(\gamma)/G_{max}$ and $\eta(\gamma)$ for sp II and III. The empirical relationships were written as a function of stress components to predict the small and intermediate strain properties of granular material subjected to stress induced anisotropy for different stress paths.

DEM analyses were also conducted on a granular packing to determine the effect of stress induced anisotropy on the contact properties for various stress paths from the microscopic point of view. The micro-mechanical observations showed that the variations in normal contact force in the grain-to-grain contacts and coordination numbers were the main micro-mechanical properties which had a significant effect on the values of $G_{max}$ and $G/G_{max}$ for different stress paths.
Zusammenfassung

In dieser Arbeit wird der Einfluss des Feinkornanteils und der spannungsinduzierten Anisotropie auf die Steifigkeit und das Dämpfungsverhalten von granularen Materialien bei kleinen und mittleren Dehnungszuständen auf makroskopischer und mikroskopischer Ebene betrachtet. Ziel ist es für die unterschiedlichen Ausgangs- und Randbedingungen (Feinkornanteil und anisotrope Spannungszustand) Ansätze zu entwickeln, die maßgebenden Steifigkeiten (Steifigkeit bei kleinen Dehnungen $G_{\text{max}}$, Steifigkeit als Funktion der Dehnung, $G(\gamma)$, und die zutreffende Dämpfung, $\eta(\gamma)$), aus möglichst wenig Versuchen zu bestimmen. Entsprechend gliedert sich die Arbeit in zwei Teile. Im ersten Teil (1) wird der Einfluss des Fein-kornanteils betrachtet, im zweiten (2) der Einfluss der spannungsinduzierten Anisotropie.

Zu (1) werden zunächst Resonant Column (RC) Versuche an Huston Sand durchgeführt, wobei der Einfluss des Spannungszustandes und der Porenzahl auf $G_{\text{max}}$, $G(\gamma)$ und $\eta(\gamma)$ erfasst wird. Anschließend wird mit dieser Versuchstechnik der Einfluss des Feinkornanteils $f_c$ in Huston Sand auf $G_{\text{max}}$, $G(\gamma)$ und $\eta(\gamma)$ systematisch in einem Bereich von $f_c = 0$ bis $f_c = 40\%$ untersucht.

Die Versuche zeigen, dass $G_{\text{max}}$ mit zunehmender Porenzahl und zunehmenden Feinkornanteilen kleiner wird. Die Dämpfung, $\eta(\gamma)$, steigt zunächst mit zunehmendem Feinkorngehalt bis $f_c = 20\%$ an. Bei größerem $f_c$ nimmt sie wieder ab. Im Mikro-CT konnte nachgewiesen werden, dass sich auf mikroskopischer Ebene zwei unterschiedliche Strukturen in Abhängigkeit des Feinkornanteils ausbilden, "Feines im Sand", wenn die feinen Partikel an der Lastabtragung zwischen den Körnern nur begrenzt partizipieren, und "Sand im Feinen", wenn die Sandkörner im Feinanteil schwimmen. Die Zustandsgröße Porenzahl $e$ kann diese Übergänge nicht abbilden, wogegen die äquivalente Porenzahl $e^*$ eine konsistente Zustandsgröße darstellt. Mit deren Einführung ergibt sich die Beziehung $G_{\text{max}}/Ap_\gamma \left(\frac{p}{p_a}\right)^n \sim f(e^*)$, worin die Parameter $A$ und $n$ unabhängig vom Feinkornanteil sind und am Sand ohne Feinkornanteil ermittelt werden können. Damit können $G_{\text{max}}$, $G(\gamma)$ und $\eta(\gamma)$ für einen Sand mit unterschiedlichem Feinkornanteil allein auf Basis von Versuchen am reinen Sand bestimmt werden.

Zu (2) wurde das vorhandene RC-Gerät zunächst um eine Belastungseinrichtung zum Aufbringen zusätzlicher Vertikalspannungen und damit anisotroper Spannungszustände auf die Probe erweitert. Dazu wurde das Systemverhalten durch numerische Berechnungen
abgebildet und das Gerät nach dem Umbau unter anderem durch Vergleichsmessungen mit Piezoelementen kalibriert und die Funktion validiert. Anschließend wurde das Verhalten von Huston Sand unter vier Spannungspfaden (sp) ermittelt: isotrope Belastung, sp I ($\sigma_3 = \text{const.}$), sp II ($p = \text{const}$), sp III ($K = \text{const}$.) mit $\sigma_1$ und $\sigma_3$ Hauptspannungen, $p = (\sigma_1 + 2\sigma_3)/3$, $K = \sigma_3/\sigma_1$. Die Abhängigkeit der Eigenschaften des Sandes bei kleinen Dehnungen vom Spannungspfad wurde dabei deutlich. Unter sp I ist die Auswirkung des Spannungsverhältnisses auf $G_{\text{max}}$ ausgeprägt aber auf $G(\gamma)$ und die Dämpfung, $\eta(\gamma)$, nicht. Bei sp II und sp III wirkt sich die Anisotropie auf alle drei Parameter, $G_{\text{max}}$, $G(\gamma)$ und $\eta(\gamma)$ aus. Bekannte empirische Beziehungen werden verwendet, um die Spannungspfadabhängigkeit von $G_{\text{max}}$, $G(\gamma)$ und $\eta(\gamma)$ zu erfassen und ein Bestimmen dieser Parameter zu ermöglichen. Um die gefundenen Abhängigkeiten physikalisch zu erklären, wird das Spannungs-Verformungsverhalten granularer Materialien bei kleinen Dehnungen numerisch mit der Methode der DEM abgebildet. Dieses erlaubt die Lastabtragung zwischen den Körnern auf der Mikrostrukturebene zu bewerten. Aus den Ergebnissen wird die Abhängigkeit zwischen der anisotropen Belastung, den Veränderungen in den Normalkräften in den Korn-zu Korn-Kontakten sowie der Koordinationszahl und den Veränderungen von $G_{\text{max}}$, $G(\gamma)$ und $\eta(\gamma)$ bei den verschiedenen Spannungspfaden deutlich.
Vorwort des Herausgebers


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Nomenclature

$A$  Constant fitting parameter in Hardin’s relation

$a$  A factor which is a function of rotational frequency and shear wave velocity

$A_1, A_2$ The cross section area of loading bar

$B$  Constant fitting parameter of Hardin’s relation with coordination number function

$b$  Fraction of active fines in sand force structure

$C$  Cycle of analysis in DEM simulation

$c_1, c_2$  Constant fitting parameters of damping ratio function

$CN$  Coordination number

$CN_{coarse-coarse}$  Coarse-coarse coordination number

$C_{uc}, C_{uf}$  Uniformity coefficient of coarse and fine mixture

$D, d$  Size of coarse and fine particles

$D_{10}$  Size of sand at 10% finer

$d_{50}$  Size of fine at 50% finer

$e$  Void ratio

$e_0$  Initial void ratio

$E_{max}$  Maximum Young’s modulus

$e_{max}, e_{min}$  Maximum and minimum void ratio

$e^*$  Equivalent granular void ratio
$F$ Generated vertical load by double acting pressure device

$f_c$ Fines content

$F_N$ Normal contact force between two particles

$f_r$ Resonant frequency

$Frac$ Fraction factor in DEM simulation

$F_T$ Shear contact force between two particles

$f_{thr}$ Threshold fines content

$F_v$ Corrected vertical load on top of the actuator

$f_1, f_2$ The value of frequency when the amplitude of excitation is equal to 0.707 time of maximum amplitude

$G_{max}$ Maximum shear modulus

$G_s$ Specific gravity

$G_p$ Shear modulus of particles

$G_1, G_2$ Shear stiffness of particles

$G_\gamma$ Shear modulus respect to shear strain

$J$ Mass moment of inertia of sample

$J_L$ Mass moment of inertia of bottom part

$J_{mass}$ Mass moment of inertia of extra mass

$J_0$ Mass moment of inertia of top part

$k$ Constant fitting factor

$K_N$ Normal stiffness between two particles

$K_T$ Tangential stiffness between two particles

$L$ Height of sample
\begin{itemize}
\item $l$ Distance between center of two particles
\item $L_1, L_2$ Constant parameters related to the particle properties
\item $m$ Factors related to the effect of coarse in fines mixture
\item $M_{\text{max}}$ Maximum P-wave modulus
\item $m_1, m_3$ Stress exponents in reference shear strain function
\item $N_c$ Number of contacts
\item $N_{c,\text{coarse-coarse}}$ Number of contacts between coarse particles
\item $N_{b,\text{coarse}}$ Number of coarse particles
\item $n$ Exponent of mean effective stresses of Hardin’s relation
\item $n_{cn}$ Exponent of coordination number function
\item $N_B$ Number of particles
\item $N_0$ Number of particles with zero contacts
\item $N_1$ Number of particles with one contacts
\item $n_1, n_2, n_3$ Exponent of stress components of Roesler’s relation
\item $P, P_0$ Applied pressures in double acting cell pressure for applying vertical stress in the Bochum RC device
\item $p'$ Mean effective stress
\item $p_a$ Atmospheric pressure
\item $p_{\text{kk1}}, p_{\text{kk2}}$ Peak to peak amplitude of signals from the resonant column
\item $Q$ Amplitude of oscillation for two particles model
\item $R$ Particle radius
\item $s$ Contact area between loading bar and actuator
\item $t$ Diameter of aluminium sample
\end{itemize}
\( v_s \)  Shear wave velocity

\( W \)  Total energy

\( x, y \)  Fitting parameters of void ratio function

\( z_1, z_2 \)  The peak to peak amplitude of recorded signals from accelerometer

\( \alpha \)  Hertz contact area between two particles

\( \beta \)  Fitting parameters of Hardin’s hyperbolic curve

\( \gamma \)  Amplitude of shear strain

\( \gamma_{d_{\text{min}}} \) and \( \gamma_{d_{\text{max}}} \)  Minimum and maximum dry density

\( \gamma_r \)  Reference shear strain

\( \gamma_{e_{\text{t}}} \)  The maximum shear strain that the shear stiffness has the maximum value

\( \gamma_{r_{1}} \)  Reference shear strain at \( \dot{\rho}=100 \) kPa

\( \delta \)  Sliding of particles with oscillation

\( \Delta t \)  Time step

\( \Delta v \)  Deviation of obtained frequency of aluminium sample from experimental

\( \Delta W \)  Dissipation of energy

\( \Delta_1, \Delta_2 \)  Deviation of obtained frequency of aluminium sample during anisotropic loading from FEM simulation

\( \dot{\epsilon} \)  Rate of boundary motion in DEM simulation

\( \zeta \)  Exponent of contact forces

\( \eta \)  Damping ratio

\( \eta_{\text{min}} \)  Minimum damping ratio

\( \kappa \)  Exponent of mean effective stress in Stokoe’s relationship

\( \theta \)  Volumetric water content
$\lambda, n_b$  Fitting parameters of $b$ function

$\mu$  Friction coefficient between two particles

$\mu_i$  Initial friction coefficient in DEM simulation

$\mu_f$  Final friction coefficient in DEM simulation

$\nu$  Poisson’s ratio of particles

$\rho$  Density of sample

$\sigma_1, \sigma_2, \sigma_3$  Stress components of Roesler’s relation

$\sigma_3$  Confining pressure inside the resonant column cell

$\tau_{max}$  Peak shear strength

$\tau_r$  Shear strength at reference shear strain

$\phi$  Friction angle

$\Omega$  Exponent of normal contact force in normal contact stiffness relationship

$\omega$  Rotational frequency
1 Introduction

1.1 Introduction

Experimental and analytical studies in the last few decades show that the mechanical properties of geo-materials strongly depend on the amplitude of deformation or vibration generated by dynamic events, e.g. earthquake, traffic, machine foundations. For example, during an earthquake, compression and shear waves propagate through the soil mass from the bedrock to the designed structures. The goal is to find the effect of waves on the stability of structures located on the soil mass. The structures are usually safe against compression waves because of the weight of the structure. Therefore, the shear component of seismic waves is usually the most critical component because of the susceptibility of structures to horizontal motions and ground settlement during vibration (Darendeli 2001). Thus, to evaluate the stability of structures, it is important to assess the response of soil to the shear wave (vertical propagation and horizontal polarization) which is generated by earthquake or vibration of bedrock (Figure 1.1). Different parameters (e.g. depth or distance of vibration source, soil properties and duration of vibration) make significant effects on response of structure or ground motion during a vibration. Among these parameters, soil properties can make significant effects on soil-structure interaction during wave propagation (Darendeli 2001). Shear modulus, $G$, and material damping, $\eta$, are soil properties which have a significant effect on ground motion during vibration (Seed et al. 1986). These properties are used to determine the velocities and decay of stress waves propagating through the geomaterials.
1 Introduction

**Figure 1.1:** Sketch of shear wave propagation in soil deposit and factors influencing ground motion

**Figure 1.2:** Schematic sketch of shear stress versus shear strain: (a) monotonic and cyclic loading (Mitchell & Soga 2005); (b) damping ratio, $\eta = \frac{\Delta W}{2\pi W}$ (Ishihara 1996)
1.1 Introduction

Furthermore, settlement of soils is one of the challenging problems in the soil-structure interaction problems during earthquake or ground shaking. Differential settlement may cause distress in the structures founded in the soil mass during earthquake (Ghayoomi et al. 2010). The primary factor controlling settlement in dry sands is cyclic shear strain amplitude during earthquake shaking which is a function of shear modulus at any given depth in a soil deposit (e.g. Tokimatsu & Seed 1987). The settlement behavior of soils during earthquake may differ from those predicted by models based on the isotropic stress conditions (e.g. Tokimatsu & Seed 1987), which can lead to an inefficient or unsafe design. Therefore, to perform an accurate dynamic analysis the shear stiffness and damping ratio must be determined accurately. Shear modulus, $G$, represents the shear stiffness of soil elements. Shear modulus can be defined as the gradient of $\tau$-$\gamma$ curve (Ishihara 1996). Regarding this curve, shear modulus of soils changes with strain amplitude (Figure 1.2a). In soil elements subjected to cyclic loading, generated hysteresis loop (Figure 1.2b) represents the effect of shear strain on shear stress (Ishihara 1996). In this case, damping of material is defined as dissipated energy, $\Delta W$, at a cycle of loading over the total energy, $W$ (Ishihara 1996). Dissipated energy over the loading cycle is represented by the white area in Figure 1.2b (hysteresis loop) and total energy is represented by triangular area (hatched area).

![Figure 1.3: Characteristics of shear modulus and damping ratio of soils at different strain levels (Wang & Santamarina 2007)](image-url)
Shear stiffness and damping ratio are significantly affected by amplitude of shear strain which is important in deposited soils because the stiffness and damping in each layer is related to the amplitude of shear strain in that layer. Figure 1.3 shows a schematic shape of damping and modulus ratio versus amplitude of shear strain in logarithmic scale. This figure shows that the shear modulus decreases with an increase in the amplitude of shear strain and damping ratio increases with an increase in the amplitude of shear strain.

The previous studies using resonant column and torsional shear device, e.g. Hardin & Drnevich (1972b), Drnevich (1978), Seed et al. (1986) and Wichtmann & Triantafyllidis (2013), confirm that shear modulus at small strain region has the maximum value which is called maximum shear modulus, $G_{\text{max}}$. At this region soil behavior is linear and soil behave nonlinear with further increase in the amplitude of shear strain (Figure 1.3). Material damping ratio at small strain region has the minimum value which is called minimum damping ratio, $\eta_{\text{min}}$ and it increases with further increase in the amplitude of shear strain. In analyzing the response of soil elements when these elements are subjected to small deformations or vibrations, elastic parameters ($G_{\text{max}}$ and $\eta_{\text{min}}$) are applicable for analyzing the response of soils. By increasing the amplitude of deformations, shear modulus, $G$, and damping ratio, $\eta$, will be different in comparison with the initial values. The mechanical properties of soils at medium strain region (Figure 1.3) is so-called intermediate strain properties in the current research work. Small and intermediate strain properties ($G_{\text{max}}$, $G(\gamma)$, $\eta(\gamma)$) of geo-materials are key parameters to analyze the mechanical response of soil elements subjected to small deformation or vibration (Benz 2007). To perform an accurate analysis, especially in deposited soils, stiffness and damping ratio in soil elements must be determined at different level of deformation or vibration accurately. For an example, equivalent linear procedure proposed by Schnable et al. (1972) and developed by Idriss & Sun (1992) is one of the concepts that is applicable to account the nonlinearity of soils using an iterative procedure to obtain values of modulus and damping.

The small and intermediate strain stiffness of soils can be measured using laboratory tests on undisturbed or disturbed soil samples, including: piezoelectric elements (e.g. Jovicic & Coop 1988, Sadek 2006 and Wang & Mok 2008), torsional shear test (e.g. Tatsuoka et al. 1978, Stokoe et al. 1999a) and resonant column device (e.g. Hardin & Drnevich 1972b, Drnevich 1978 and Wichtmann & Triantafyllidis 2013) or in the field from in-situ tests including: seismic piezo-cone sounding (e.g. Schneider & Mayne 1999 and Stokoe et al. 1999a). In-situ geo-physical tests are also available to measure the wave velocity and consequently, stiffness in different soil layers (e.g. Figure 1.4 from Schneider & Mayne 1999).
1.2 Background and objective of this study

In the last few decades, a number of cyclic and dynamic laboratory measurements have been conducted for determining the stress-strain properties of granular arrays and natural sands at small to intermediate strain regions. The small and intermediate strain properties include: i) maximum shear modulus at small strains, $G_{\text{max}}$; ii) the variation of shear modulus with shear strain, $G(\gamma)$; iii) the variation of damping ratio with strain, $\eta$; and, iv) the reference shear strain, $\gamma_r$.

The effect of various parameters on the small and intermediate strain properties of geo-
materials have been studied in previous works. However, by reviewing the literature, systematic studies on the effect of fines content and stress induced anisotropy are rare and the following gaps can be highlighted.

Part 1: there are soils with the same host sand but with different amount of fines along the geological profile, e.g. Christchurch, New Zealand (Green & Cubrinovski 2010) and Ahmedabad, India (Raju et al. 2004) where both sites severely liquefied during earthquakes. The influence of fine particles on sand force structure, from micro-structure point of view, was presented by Mitchell (1976) and then simplified by Thevanayagam (1998) for other mechanical responses of transition soils. In transition soils, the concept of threshold fines content \( f_{thr} \) is important for separating ”fines-in-sand” and ”sand-in-fines” models (Rahman & Lo 2008). Depending on the interaction between fine and sand particles, Thevanayagam (1998) suggested an ”equivalent” void ratio, \( e^* \) instead of \( e \) as an appropriate density index (the concept of \( e^* \) is discussed later). Thevanayagam (1999) and Rahman et al. (2012) used the concept of equivalent void ratio for limited published data sets to predict maximum shear modulus in transition soils. However, collected data sets were originally not designed for the evaluation of \( e^* \) to capture the effect of \( f_c \) on \( G_{max} \) and the maximum \( f_c \) was only 20%. Thus, a suitable approach for capturing the effect of a wide range of \( f_c \) on \( G_{max} \), \( G(\gamma) \) and \( \eta(\gamma) \) is still a topic of research interest. Furthermore, the effect of \( f_c \) on the fabric of soil must be explained through the microscopic observations.

Part 2: from the literature, numerous experimental studies have been done by resonant column device to evaluate the influence of various parameters on the small strain properties of geo-materials but, these studies were restricted to the isotropic stress state. As we know, in soil-structure interaction problems, most of the times we are dealing with soil elements subjected to anisotropic loading. The small and intermediate strain properties of soil elements may be different than the observed results at the laboratory under the isotropic stress state.

Studies on the effect of stress induced anisotropy can be divided into two main groups: 1- studies on the small strain properties \( (G_{max} \) and \( \eta_{min} \)) and, 2- studies on intermediate strain properties of granular materials \( (G(\gamma) \) and \( \eta(\gamma) \)).

By reviewing the literature (Chapter 2), numerous experimental studies were conducted by piezoelectric elements for evaluating the effect of induced anisotropy on \( G_{max} \) (small strain properties) of granular materials, although some of them (e.g. Hardin & Black 1966, Tatsuoka et al. 1979, Yu & Richart 1984, Santamarina & Cascante 1996) used the resonant column device and they concluded that at stress ratio less than two, the effect of stress induced anisotropy on \( G_{max} \) is not significant but their studies were restricted to dense samples. Therefore, the effect of density of sample on the maximum shear modulus
must be assessed for different stress paths.

Regarding the literature, studies on the effect of induced anisotropy on intermediate strain properties of granular material are rare, and we can refer to the performed study by Tat-suoka et al. (1979) using a torsional shear device. He studied the effect of stress induced anisotropy on damping ratio and shear modulus for one stress path (confining pressure constant and vertical pressure variable) and reported that the impact of stress ratio on damping is not significant. But, their study was restricted to the stress path with constant confining pressure, however, soil elements may be subjected to more complicated stress conditions in comparison with the stress conditions applied in the existing studies up to now. Therefore, the second goal of this study is to find the effect of stress induced anisotropy, for different stress paths, and density of sample on the small and especially intermediate strain properties of sand. The experimental results are used to present a model for prediction the modulus degradation and damping ratio in the soil samples subjected to stress induced anisotropy for different stress paths.

Experimental methods to measure the small strain stiffness do not cause fabric changes (Santamarina & Aloufi 1999). Therefore, contact parameters, contact forces and contact numbers, will control the response of granular packing at macroscopic level. At microscopic point of view, properties of the particles, interaction laws at the contact level, as well as the distribution of contact- and force-networks, i.e. the micro-structure of the sample, can induce modifications to the shear stiffness of the packing, as measured in experiments.

In this work the discrete element method (DEM) is used to study the behavior of the force/contact network during axisymmetric compression with different stress ratio. The orientation of contacts and distribution of forces and also their evolution during the various deformation paths will be discussed. Final goal is to link the modulus in granular materials to the orientation of the normal and tangential force network. Finally, the influence of stress ratio and force network on the maximum shear modulus, modulus degradation and damping is carefully discussed and findings are compared with laboratory results from the resonant column device.

Therefore, the goals of the study are to answer the following questions:

- What is the effect of fines content,”fines-in-sand” and ”sand-in-fines”, on small and intermediate strain properties of sand samples?

- Is it possible to predict the small strain properties of host sand containing fines content with concept of equivalent granular void ratio?

- What is the effect of stress induced anisotropy for various stress paths on small and intermediate strain properties of sand?
• Is it possible to predict the small and intermediate strain properties of sand subjected to stress induced anisotropy?

• How small strain stiffness is affected by fines content and stress induced anisotropy from the microscopic point of view?

1.3 General concept of the study

Experimental and analytical methods are adopted to find and discuss the effect of fines content and stress induced anisotropy on the small and intermediate strain properties of granular materials. Resonant column device and piezoelectric elements are used for the experimental study. Afterward, Micro-CT scanning and DEM simulation are employed to analyze the experimental results from the microscopic point of view. Finally, the experimental results are used to modify and develop the empirical relationships for predicting the small and intermediate strain properties of granular materials containing fines content and subjected to stress induced anisotropy for different stress paths (Figure 1.5).

Fines content:
Resonant column and piezoelectric element tests will be conducted on the mixture containing wide range of fines content. Two mixtures will be used for this section: i) angular sand mixed with quartz powder ii) highly rounded glass particles mixed with rounded fines glass beads. Micro-CT scanning will be used to discuss the observed experimental results from microscopic point of view. Afterward, the conceptual framework developed by Thevanayagam (1998) will be assessed to predict the small strain properties of sand in fine and fine in sand mixtures.

Stress induced anisotropy:
Bochum resonant column device will be modified to perform the test on the samples subjected to the stress induced anisotropy for different stress paths. The method of modification and adopted numerical and experimental methods for validation and calibration of modified resonant column device will be discussed in this thesis. Afterward, a series of stress induced anisotropic tests will be conducted on spherical glass particles and angular sand. DEM simulations, using TRUBAL code, will be calibrated with the experimental results on glass bead samples. This analysis will be used to discuss the observed experimental results from microscopic point of view. Then, resonant column tests on sand samples will be used to develop an empirical relation to predict the intermediate strain properties of granular materials. Due to the limitation of device, maximum applied shear strain was about $3 \times 10^{-4}$ and maximum applied stress ratio was 3.00.
1.3 General concept of the study

Micro and Macro mechanical assessment of small and intermediate strain properties of soils:

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</tr>
<tr>
<td></td>
<td>Micro-CT scanning</td>
</tr>
<tr>
<td></td>
<td>Microscopic</td>
</tr>
<tr>
<td></td>
<td>Microscopic</td>
</tr>
</tbody>
</table>

Figure 1.5: Flowchart of experimental work in the current study
1.4 Organization of the dissertation

The thesis consists of seven chapters, of which the content can be briefly described as follows:

Chapter 1: Presents an introduction on this research work.

Chapter 2: Presents a brief literature review on the small strain response of geomaterials, effects of initial conditions, effects of boundary conditions and material properties. This chapter will highlight the performed works on the small strain stiffness, regarding the objective and scope of this research work.

Chapter 3: The experimental program will be discussed in this chapter. This chapter will highlight the adopted procedure and limitations of adopted procedure for approaching to the objective. Calibration, validation and limitation of adopted experimental apparatus (Bochum resonant column device) will be discussed in this chapter.

Chapter 4: Presents the observed experimental results on impact of fine particles on small strain properties of granular materials. Equivalent void ratio concept will be used to discuss the observed experimental results. Furthermore, observed experimental results will be discussed from the microscopic point of view.

Chapter 5: Discrete Element Method, DEM, will be introduced and calibrated in this chapter. Calibrated model will be used to explain the effect of stress induced anisotropy from micro-mechanic point of view.

The experimental results on the effect of stress induced anisotropy on the small strain properties of granular materials will be presented in this chapter. Empirical relations will be modified to predict the maximum shear modulus and damping of sands subjected to anisotropic loading.

Chapter 6: Draws conclusions, gives recommendations, and suggests further areas of study.
2 Literature review

2.1 Introduction

In the past few decades, a number of experimental studies have been conducted for determining the stress-strain properties of granular packings and natural sands at small to intermediate strain regions. The small and intermediate strain properties have included:

i) maximum shear modulus at small strains, $G_{\text{max}}$;

ii) the variation of shear modulus with shear strain, $G(\gamma)$;

iii) the variation of damping ratio with strain, $\eta(\gamma)$; and,

iv) the reference shear strain, $\gamma_r$.

Important experimental results have been presented by Hardin & Richart (1963), Hardin & Black (1966), Hardin & Drnevich (1972b), Afifi & Richart (1973), Iwasaki & Tatsuoka (1977), Drnevich (1978), Kokusho (1980), Avramidis & Saxena (1990), Ishibashi et al. (1991), Ashmawy & Drnevich (1994), Jamiołkowski et al. (1995), Cascante & Santamarina (1996), Lo Presti et al. (1997), Chien & Oh (1998), Fioravante (2000), Darendeli (2001), Cascante et al. (2005), Chang et al. (2006), Wichtmann & Triantafyllidis (2009), Clayton (2011) and Wichtmann & Triantafyllidis (2013). The published results indicate that the small and intermediate strain properties of soils are affected by various parameters, e.g. strain amplitude, stress conditions, void ratio, number of loading cycles, degree of saturation, the over-consolidation ratio (OCR), loading frequency, aging, soil gradation, soil structure, and so on. Among these parameters, the mean effective stress, void ratio and shear strain amplitude are the predominant ones.

The effects of some important parameters, including; particle characteristics, void ratio, confining pressure, fines content, sample preparation and stress induced anisotropy, on the small and intermediate strain properties of geo-materials are presented in this chapter. The effects are discussed from the micro-mechanics point of view (at particle contact level), which has a strong connection to particle and contact characteristics. The gaps in our current understanding on the effects of fines content and stress induced anisotropy will be highlighted in this chapter.
2.2 Particle characteristics

Particle characteristics include particle size, particle shape, and mineral composition of particles. From a microscopic point of view, Skinner (1969) performed a series of shear box tests and showed that the coefficient of friction between two spherical glass ballotini increases with an increase in the particle size. From a macroscopic point of view, in spherical arrays, Patel et al. (2008) conducted tests using bender elements and reported that the size dependence of the small strain stiffness is not in line with the results presented for sand samples (Figure 2.1a). However, it appears to be consistent with the results presented by Bartake & Singh (2007), who performed BE tests on three dry sands with similar gradation and found $G_{\text{max}}$ increased as the mean particle size of the sand decreased. Gu & Yang (2013), conducted a series of RC and BE tests on glass beads with different particle sizes and concluded that the maximum shear modulus is affected slightly by particle size and $G_{\text{max}}$ decreased with increasing particle size in glass bead samples. From the micro-mechanic perspective, Gu & Yang (2013) concluded that the maximum shear modulus is independent of sphere radius.

Hardin & Drnevich (1972a) concluded from the results of resonant column tests that the influence of particle size on maximum shear modulus can be explained through the effect of particle size on void ratio. Ishihara (1996) summarized the published data points as in Figure 2.2 to show the impact of particle size. He concluded that $G_{\text{max}}$ decreases with decreasing particle size in the soil samples. Hardin & Kalinski (2005), based on experimental results from the resonant column test, concluded that the value of $G_{\text{max}}$ for relatively clean uniform and graded gravels increases with particle size and they defined the particle size function ($f(D)$) to capture the impact of particle size.

![Figure 2.1](image1.png)

Figure 2.1: The effect of particle size on: (a) shear wave velocity (Patel et al. 2008); (b) maximum shear modulus (Gu & Yang 2013)
The mineral composition of particles is the primary parameter governing many properties of particles. For example, mineral composition controls the specific gravity (Terzaghi et al. 1996) and the particle elastic constants, i.e. Young’s modulus, shear modulus, and Poisson’s ratio (Mitchell & Soga 2005). Furthermore, the elastic constants influence contact properties, such as contact area, contact pressure, and consequently, contact stiffness (Hertz 1882). Mineral composition is also a key parameter influencing particle surface properties such as water adsorption/absorption, and electro-chemical bonding, especially for fine particles such as clay (Mitchell & Soga 2005). Many researchers (e.g. Koerner 1970, Procter & Barton 1974) have suggested that the particle mineral composition significantly influences the inter-particle friction coefficient.
Particle shape is an inherent soil characteristic that plays a major role in the mechanical behaviour of soils (Mitchell & Soga 2005). Barrett (1980) suggested that particle shape
includes three independent properties, particle form, particle roundness, and surface texture or particle roughness. Koerner (1970), Miura et al. (1997), Cho et al. (2006) and Reddy (2008) have shown that the void ratio range, $e_{\text{max}} - e_{\text{min}}$, is a function of particle form and particle roundness/angularity. Bowden & Tabor (1950) have suggested that particle roughness strongly influences inter-particle friction. It is recognized that particle shape controls the fabric of soils, e.g. particle and contact orientations and type of contacts (Mitchell & Soga 2005).

Cho et al. (2006) mounted piezoelectric elements in an oedometer device and studied the influence of particle shapes on shear wave velocity and they reported that the increase in the particle irregularity (angularity and/or eccentricity) leads to a decrease in stiffness (Figure 2.3). Shin & Santamarina (2013) studied the influence of particle shape on the mechanical behaviour of soils. They measured wave velocity in mixtures containing rounded Ottawa 20-30 sand and angular blasting sand using bender elements installed in the oedometer cell. From the experimental results, they reported that $G_{\text{max}}$ increased by increasing the mass fraction of angular particles.

Yang & Wei 2012 performed a series of undrained triaxial tests on the mixtures containing angular and rounded coarse and fine particles. They concluded that the behaviour of mixtures strongly depends on the shape of coarse and fine particles.

### 2.3 Void ratio

Void ratio is a packing characteristic and has a significant effect on the maximum shear modulus. From a micro-mechanics point of view, the effect of void ratio can be explained through the number of contacts (Figure 2.4). As can be seen in Figure 2.4, the number of contacts decreases significantly with an increase in the void ratio. The effect of void ratio on the maximum shear modulus can be explained through the coordination number, $CN$, from the micro-mechanic points of view (Section 5.2). It has been shown that $CN$ strongly depends on the void ratio, e.g. Smith et al. (1929), Field (1963), Oda (1972), Oda (1977), Yanagisawa (1983) and so on.

At a macroscopic level, experimental evidence confirms that the value of $G_{\text{max}}$ decreases with an increasing void ratio (Figure 2.5). The impact of void ratio on maximum shear modulus is captured through void ratio function ($f(e)$) which was developed by Hardin & Drnevich (1972b). Other researchers in the past few decades, in relation to their experimental results, have proposed various void ratio functions which are summarized in
Table 2.1. In contrast, the curves $G(\gamma)$ and $\eta(\gamma)$ are rather independent of soil density or void ratio (e.g. Tatsuoka et al. 1978, Kokusho 1980, Wichtmann & Triantafyllidis 2013).

![Figure 2.4: Contact number versus void ratio, modified after Yanagisawa (1983)](image1)

Figure 2.4: Contact number versus void ratio, modified after Yanagisawa (1983)

![Figure 2.5: Maximum shear modulus versus void ratio from published experimental results, modified after Bui (2009)](image2)

Figure 2.5: Maximum shear modulus versus void ratio from published experimental results, modified after Bui (2009)
Table 2.1: Some of the main void ratio functions from the literature, modified after Bui (2009)

<table>
<thead>
<tr>
<th>Soil</th>
<th>e</th>
<th>( f(e) )</th>
<th>n</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ottawa sand</td>
<td>-</td>
<td>( \frac{(2.174 - e)^2}{1 + e} )</td>
<td>0.5</td>
<td>Hardin &amp; Black (1966)</td>
</tr>
<tr>
<td>Crusged Quartz</td>
<td>-</td>
<td>( \frac{(2.973 - e)^2}{1 + e} )</td>
<td>0.5</td>
<td>Hardin &amp; Black (1966)</td>
</tr>
<tr>
<td>Kaolinite</td>
<td>-</td>
<td>( \frac{(2.97 - e)^2}{1 + e} )</td>
<td>0.5</td>
<td>Hardin &amp; Drnevich (1972b)</td>
</tr>
<tr>
<td>Bentonite</td>
<td>1.5-2.5</td>
<td>( \frac{(4.4 - e)^2}{1 + e} )</td>
<td>0.5</td>
<td>Marcuson &amp; Whals (1972)</td>
</tr>
<tr>
<td>Teganuma Clay</td>
<td>1.5-4</td>
<td>( \frac{(7.32 - e)^2}{1 + e} )</td>
<td>0.45</td>
<td>Kokusho et al. (1982)</td>
</tr>
<tr>
<td>Quiou sand</td>
<td>0.9-1.1</td>
<td>( e^{-1.3} )</td>
<td>0.62</td>
<td>Lo Presti et al. (1997)</td>
</tr>
<tr>
<td>Toyoura sand</td>
<td>0.81-0.95</td>
<td>( e^{-1.3} )</td>
<td></td>
<td>Lo Presti et al. (1997)</td>
</tr>
<tr>
<td>Ticino sand</td>
<td>0.81-0.95</td>
<td>( \frac{(2.27 - e)^2}{1 + e} )</td>
<td>0.43</td>
<td>Lo Presti et al. (1993)</td>
</tr>
<tr>
<td>reconstituted clay</td>
<td>-</td>
<td>( (1 + e)^{-2.4} )</td>
<td>0.5</td>
<td>Shibuya et al. (1997)</td>
</tr>
<tr>
<td>Kenya carbonate sand</td>
<td>0.81</td>
<td>( e^{-0.8} )</td>
<td>0.516</td>
<td>Fioravante (2000)</td>
</tr>
<tr>
<td>Quartz sand</td>
<td>0.4-0.85</td>
<td>( \frac{(1.8 - e)^2}{1 + e} )</td>
<td>0.42</td>
<td>Wichtmann &amp; Triantafyllidis (2009)</td>
</tr>
<tr>
<td>Glass beads</td>
<td>0.58-0.65</td>
<td>( e^{-3.98} )</td>
<td>0.403</td>
<td>Goudarzy et al. (2014)</td>
</tr>
</tbody>
</table>

2.4 Confining pressure

Stiffness of granular materials depends on the micro-structural properties, mainly, particle and contact properties. However, for a given material, particle properties may remain the same and contact properties will have the major contribution to the variation in packing stiffness. According to the Hertz-Mindlin’s theory, the normal and tangential stiffness are function of contact forces and the elastic properties of a particle (Johnson 1985 and Yimsiri & Soga 2002).

\[
K_T = L_2 K_N \left[ 1 - \frac{F_T}{F_N \tan \phi} \right]^\zeta
\]  

(2.1)

Where \( K_N \) is the normal stiffness (\( K_N = L_1 F_N^{\Omega} \)), \( L_1 \) and \( L_2 \) are related to the particle properties, \( \zeta \) and \( \Omega \) are material constants (\( \zeta = \Omega = 1/3 \), Yimsiri & Soga 2002), \( F_N \) and \( F_T \) are normal and tangential forces at contact points, and \( \phi \) is the friction angle between two particles.

The effect of confining pressure on the distribution of contact forces have also been studied in recent decades (e.g. Petrakis et al. 1988, Rothenburg & Bathurst 1989, Ng & Petrakis 1996, Wang & Mok 2008).

Several authors (e.g. Gassmann 1951, White & Sengbush 1953, Brandt 1955) calculated
the influence of isotropic confining pressure on wave velocity for various arrays of smooth and rough spheres. Also, they concluded that both velocities increase proportionally to $\sigma_0^{1/6}$. Based on Hertz’s theory, Duffy & Mindlin (1957) and Duffy (1959) theoretically concluded that the tangential stiffness of sphere packing is proportional to the cubic root of effective stress ($\sigma^{1/3}$). For regular packing, Petrakis & Dobry (1987), and for random packing of sphere, Walton (1987) and Liao et al. (2000) have shown that the exponent of isotropic pressure is equal to 1/3.

At a macroscopic level, it is well-known that $G$ is related to $\sigma^n$; the exponent $n$ can be used as an indicator of the type of contact which can be a value from 1/3 for spherical contacts to 1/2 for cone to plane contacts (Cascante & Santamarina 1996). Experimental works conducted by many researchers with various soils show that $G_{\text{max}}$ increases in an exponential manner with mean effective stress. A summary of determined values for stress exponent, $n$, is presented in Table 2.1. This table shows the exponent $n$ in practice varies from 0.4 to 0.62 and the value of 0.5 was proposed by many researchers.

From a microscopic point of view, dissipation of energy between two particles (Figure 2.6a) subjected to the sinusoidal oscillation is written as Equation 2.2 (Johnson 1985).

$$\Delta W = \frac{9\mu^2F_N^2}{10R} \left( \frac{2 - \nu_1}{G_1} + \frac{2 - \nu_2}{G_2} \right) \left[ 1 - \left( 1 - \frac{Q}{\mu F_N} \right)^{5/3} - \frac{5Q}{6\mu F_N} \left[ 1 - \left( 1 - \frac{Q}{\mu F_N} \right)^{2/3} \right] \right]$$

(2.2)

When the amplitude of oscillation is too small ($\mu F_N >> Q$), the Equation 2.2 is summarized to Equation 2.3 (Johnson 1985).

$$\Delta W = \frac{1}{36R\mu F_N} \left( \frac{2 - \nu_1}{G_1} + \frac{2 - \nu_2}{G_2} \right) Q^3$$

(2.3)

where, $G_1$ and $G_2$ are shear stiffness of particles, $\nu_1$ and $\nu_2$ are Poisson’s ratio of particles, $\mu$ is the friction coefficient between particles, $F_N$ is the normal contact force between particles, $Q$ is the amplitude of oscillation, $R$ is the particle radius. Damping ratio is defined as dissipated energy ($\Delta W$) at one cycle of loading over the total energy ($W$), $\eta = \frac{\Delta W}{2\pi W}$, (Ishihara 1996). For two particles with the same properties, total energy ($W$) is the area of the hatched part in Figure 2.6b, $W = (Q \times \delta)/2$. Equation 2.3 was used to determine the dissipated energy between two particles subjected to a low amplitude of oscillation. Then damping between two particles, with the same properties, subjected to oscillation, can be determined using Equation 2.4 (Bui 2009).
2.4 Confining pressure

The relative tangential displacement of the two particles (Figure 2.6a) is found by Equation 2.5 (Johnson 1985).

\[
\delta = \frac{3\mu F_N}{16a} \left( \frac{2-\nu_1}{G_1} + \frac{2-\nu_2}{G_2} \right) \left[ 1 - \left(1 - \frac{Q}{\mu F_N} \right)^{2/3} \right] \tag{2.5}
\]

where, \(a\) is the Hertz contact area. With regard to Figure 2.6a, shear strain for two particles with the same properties is equal to \(\gamma = \delta/l\). Thus, from Equation 2.5 the value of \(\gamma\) can be determined by Equation 2.6 (Bui 2009).

\[
\gamma = \frac{3\mu F_N}{8al} \left( \frac{2-\nu}{G} \right) \left[ 1 - \left(1 - \frac{Q}{\mu F_N} \right)^{2/3} \right] \tag{2.6}
\]

From the presented relations at a microscopic level, it can be concluded that particle characteristics, contact force and amplitude of oscillation are factors influencing the damping ratio in the small strain region.

Experimental works confirm that damping ratio is significantly affected by mean effective stress at macroscopic level (e.g. Tatsuoka et al. 1978, Kokusho 1980 and Wichtmann & Triantafyllidis 2013). The experimental results show that damping ratio decreases with an increase in the mean effective stress.
2.5 Empirical relationships

Research efforts, since 1960’s, have advanced our understanding of the factors and how they affect $G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$ which has led to the development of various prediction models which are applicable to the computer programs for site response analysis. Earlier studies recognized that $G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$ for soils depend on the packing density and mean effective stress (Hardin & Black 1966, Hardin & Drnevich 1972b, Drnevich 1978, Tatsuoka et al. 1978, Seed et al. 1984). From the experimental results using the resonant column device, Hardin & Black (1966) proposed one of the most widely used empirical relationship that considers the effect of density through void ratio ($e$) and $p'$ to predict $G_{\text{max}}$ of a soil. This relationship is referred as Hardin’s relation which is represented by the following general form (Equation 2.7):

$$G_{\text{max}} = Ap_a\left(\frac{p'}{p_a}\right)^n f(e)$$

(2.7)

where, $A$ is a material constant which depends on soil type, $p_a$ is atmospheric pressure (100 kPa), $p'$ is mean effective stress, $n$ is an exponent and $f(e)$ is the void ratio function. Seed et al. (1984) also proposed a relationship between $G_{\text{max}}$ and $p'$ as $G_{\text{max}}=218.8 K_{2,\text{max}}(p')^{0.5}$ (in SI unit), where $K_{2,\text{max}}$ may be a function of $e$. Although the above relationships were adequate to predict $G_{\text{max}}$ for a particular soil, the evaluation of their relative performance with a large data set is rare. Also, the effect of particle shape, size and their distribution for different type of soils were not considered in the above relationships. Recent studies show that particle size and their distribution have a significant effect on $G_{\text{max}}$ (Darendeli 2001, Menq 2003, Hardin & Kalinski 2005, Wichtmann & Triantafyllidis 2009, Wichtmann & Triantafyllidis 2013 and Wichtmann & Triantafyllidis 2014). Wichtmann & Triantafyllidis (2009) studied the effect of grain size distribution on small and intermediate strain properties of sand samples using the Bochum resonant column device. They suggested a correlation for fitting parameters of Hardin’s relation to $C_u$ and $d_{50}$. Hardin & Kalinski (2005) studied the influence of particle size and reported that $G_{\text{max}}$ increased with particles size for uniform graded gravelly sands and Hardin’s relation can be a function of particle size, $f(D)$.

Empirical relationships were also developed by Hardin & Black (1966) and Roesler (1979) to predict the maximum shear modulus ($G_{\text{max}}$) in the samples subjected to stress induced anisotropic conditions. Hardin & Black (1966) believed that shear stress has an insignificant effect on shear modulus, and thus concluded that Equation 2.8 can be used to predict the maximum shear modulus with sufficient accuracy.

$$G_{\text{max}} = Af(e)p_a^{(1-n)}\left(\frac{\sigma_1 + \sigma_3}{2}\right)^n$$

(2.8)
where, \( A \) is the dimensionless coefficient, \( f(e) \) is a void ratio function, \( n \) is the empirical stress exponent and \( p_a \) is atmospheric pressure. Roesler (1979) performed experimental studies on cubic soil samples to evaluate the effects of stress components on shear wave velocity under anisotropic loading. Roesler (1979) modified general Hardin’s relation based on the stress components (Equation 2.9).

\[
G_{\text{max}} = A f(e) p_a \left( \frac{\sigma_1}{p_a} \right)^{n_1} \left( \frac{\sigma_2}{p_a} \right)^{n_2} \left( \frac{\sigma_3}{p_a} \right)^{n_3}
\]  

(2.9)

where, \( n_1 \) and \( n_3 \) are the empirical stress exponents for vertical and horizontal stress respectively; \( \sigma_2 \) is horizontal stress (out of plan stress) and \( p_a \) is atmospheric pressure. Based on the studies conducted by Bellotti et al. (1996) and Sadek et al. (2007), \( n_2 \) is approximately zero. Thus, Roesler’s relation can be applied for cylindrical samples in a resonant column device. A Summary of determined values for stress exponents of sample subjected to stress induced anisotropic tests is presented in Table 2.2.

After Roesler (1979), Yu & Richart (1984) studied the effects of the stress ratio on the maximum shear modulus (\( G_{\text{max}} \)) of soils subjected to a stress ratio of more than two. From the observed experimental results, they proposed a reduction factor for Roesler’s and Hardin’s relationships.

Empirical relationships were also developed and modified to predict the modulus degradation and damping of granular materials.

### Table 2.2: The obtained fitting parameters for relationships 2.8, 2.9 from previous works, (Modified after Wang & Mok 2008)

<table>
<thead>
<tr>
<th>Test Method</th>
<th>Material</th>
<th>( n )</th>
<th>( n_1 )</th>
<th>( n_3 )</th>
<th>Researcher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cubical sample</td>
<td>Fine sand</td>
<td>0.512</td>
<td>0.298</td>
<td>0.214</td>
<td>Roesler (1979)</td>
</tr>
<tr>
<td>Resonant column</td>
<td>Ottawa sand</td>
<td>-</td>
<td>0.25</td>
<td>0.28</td>
<td>Yu &amp; Richart (1984)</td>
</tr>
<tr>
<td></td>
<td>Brazil sand</td>
<td>-</td>
<td>0.24</td>
<td>0.28</td>
<td></td>
</tr>
<tr>
<td>Cylindrical chamber</td>
<td>Ticino sand</td>
<td>0.54</td>
<td>0.24</td>
<td>0.30</td>
<td>Lo Presti &amp; O’Neill (1991)</td>
</tr>
<tr>
<td>Resonat column</td>
<td>Barco sand</td>
<td>0.54</td>
<td>0.20</td>
<td>0.34</td>
<td>Santamarina &amp; Cascante (1996)</td>
</tr>
<tr>
<td>Triaxial specimen</td>
<td>Ticino sand</td>
<td>0.44</td>
<td>0.272</td>
<td>0.168</td>
<td>Fioravante (2000)</td>
</tr>
<tr>
<td></td>
<td>Kenya sand</td>
<td>0.52</td>
<td>0.244</td>
<td>0.272</td>
<td></td>
</tr>
<tr>
<td>True triaxial box</td>
<td>Toyoura sand</td>
<td>0.483</td>
<td>0.27</td>
<td>0.27</td>
<td>Wang &amp; Mok (2008)</td>
</tr>
</tbody>
</table>
Hardin & Drnevich (1972a) proposed a well known empirical relationship (Equation 2.10) to predict the backbone curve in nonlinear behavior of soil elements. Hardin’s relation is based on two main parts: maximum shear modulus and reference shear strain.

\[
\frac{G}{G_{\text{max}}} = \frac{1}{1 + \frac{\gamma}{\gamma_r}}
\]  

(2.10)

where \(G_{\text{max}}\) is the maximum shear modulus, \(\gamma\) is shear strain and \(\gamma_r\) is the reference shear strain. While \(\gamma_r\) determined at \(G/G_{\text{max}} = 0.5\) would be the most objective way, the stiffness curves obtained from the resonant column tests were in the range of \(G/G_{\text{max}}\) of 0.68 to 1.0 for clean sand, which would not give a reliable \(\gamma_r\). However, \(\gamma_r\) can be estimated from Equation 2.11:

\[
\gamma_r = \frac{\tau_r}{G}
\]  

(2.11)

where, \(\tau_r\) is the shear stress corresponding to \(\gamma_r\) at \(G/G_{\text{max}}\) equal to 0.5.

Also, Hardin & Drnevich (1972a) proposed Equation 2.12 to predict the modulus degradation.

\[
\frac{G}{G_{\text{max}}} = \frac{1}{1 + \frac{\gamma}{\gamma_r}[1 + \beta \exp(-\frac{\gamma}{\gamma_r})]}
\]  

(2.12)

where \(\alpha\) and \(b\) are fitting curve parameters and \(\gamma_r\) is the reference shear strain.

The value of reference shear strain is defined as (Hardin & Drnevich 1972a):

\[
\gamma_r = \frac{\tau_{\text{max}}}{G_{\text{max}}}
\]  

(2.13)

where, \(\tau_{\text{max}}\) is the peak shear strength which is determined by Equation 2.14 and \(G_{\text{max}}\) is the maximum shear modulus (Hardin & Drnevich 1972a).

\[
\tau_{\text{max}} = \sigma_v \left[ \left( \frac{1 + \left( \frac{\sigma_3}{\sigma_1} \right)}{2} \sin \phi \right)^2 - \left( 1 - \frac{\sigma_3}{\sigma_1} \right)^2 \right]^{1/2}
\]  

(2.14)

where, \(\phi\) is the friction angle, \(\sigma_3\) and \(\sigma_1\) are the confining pressure and the vertical stress respectively. Tatsuoka et al. (1979) showed that Equation 2.13 and Equation 2.14 are not valid for determining the reference shear strain in samples subjected to stress induced anisotropy.

The dependency of \(\gamma_r\) on the confining pressure can be described as (Stokoe et al. 1999b):

\[
\gamma_r = \gamma_{r1} \left( \frac{p'}{p_a} \right)^\kappa
\]  

(2.15)

where, \(\gamma_{r1}\) is the reference shear strain when the confining pressure is equal to 100 kPa, \(p_a\) is the atmospheric pressure (assumed 100 kPa) and \(\kappa\) is the stress exponent.
Additionally, in the cases that there is not information about the shear strength of materials, the normalization of $\gamma$ can be done by $\frac{\sqrt{p}}{\gamma_{pa}}$ instead of $\gamma_r$ which is valid for soil elements subjected to isotropic loading (Hardin & Kalinski 2005). Furthermore, damping can be written as a function of modulus degradation (Zhang et al. 2005).

$$\eta - \eta_{\text{min}} = c_1 \left( \frac{G}{G_{\text{max}}} \right)^2 - c_2 \frac{G}{G_{\text{max}}} + (c_2 - c_1)$$

(2.16)

where $c_1$ and $c_2$ are constant parameters and $\eta_{\text{min}}$ is the minimum damping at small strain region. Following a review of the literature we can see that a systematic study is essential to evaluate the effects of stress ratio and stress path on modulus degradation and damping in granular materials. Furthermore, the empirical relationships must be assessed to predict the modulus degradation and reference strain ($\gamma_r$) in soil elements subjected to anisotropic loading.

### 2.6 Fines content

Previous studies have been mainly confined to either clean sands or gravel. Systematic studies on transition soils (i.e. clean sand mixed with fine particle, $d \leq 0.075\text{mm}$) containing a wide range of fines content are rare, although transition soils are not uncommon. A recent example of such transition soil is the many liquefied sites in Christchurch, New Zealand, due to the Darfield earthquake (2010) where clean sand has almost the same grain size distribution but is mixed with different percentage of fines content, $f_c$ (Green & Cubrinovski 2010).

The effect of fine particles on sand force structure, from a micro-structure point of view, was presented by Mitchell (1976) and then simplified by Thevanayagam (1998) for other mechanical responses of transition soils. When fine particles are sufficiently smaller than sand particles, all fine particles do not contribute in sand force structure when loaded. Thus, by neglecting some fine particles in sand force structure, Thevanayagam (1998) suggested an "equivalent void ratio", $e^*$, instead of $e$ as an appropriate density index (the concept of $e^*$ is discussed later). Thevanayagam & Liang (2001) assumed secant modulus at axial strain of 0.0005 from a triaxial test as $G_{\text{max}}$ for Foundary Sand with up to 25% non-plastic $f_c$ (fines with PI=0), and reported that the effect of $f_c$ can be captured by replacing $e$ by $e^*$ in Equation 2.7. However, the process of obtaining $e^*$, during that time, was essentially a back analysis process which limits the applicability of Hardin’s relation (Equation 2.7). Rahman et al. (2012) estimated $e^*$ from soil grading properties and used...
this in Hardin’s relation to capture the effect of fines for collected data sets. They reported that $e^*$ in Equation 2.7 can capture the effect of non-plastic fines. However, collected data sets were not designed for the evaluation of $e^*$ to capture the effect of $f_c$ on $G_{\text{max}}$ and the maximum $f_c$ covered was only 20%. Thus, a suitable approach for capturing the effect of non-plastic $f_c$ on $G_{\text{max}}$ is still a topic of research interest.

From a macroscopic point of view, a systematic study on the effect of fine particles on $G_{\text{max}}$ was first presented by Iwasaki & Tatsuoka (1977). Two clean sands, Iruma Z1 and Iruma W, were mixed with a different percentage of non-plastic $f_c$ and resonant column tests were conducted to assess the effect of fines content (Figure 2.7). With regard to the experimental results, Iwasaki & Tatsuoka (1977) concluded that the value of $G_{\text{max}}$, obtained from resonant column test, decreased with an increase in $f_c$ (Figure 2.7). Based on the bender element tests, Salgado et al. (2000) also reported that $G_{\text{max}}$ for Ottawa sand decreased with an increase in non-plastic $f_c$. Tao et al. (2004) performed cyclic torsional shear tests on thin hollow cylindrical samples to evaluate the effect of fines content on the stress-strain behaviour of sands obtained from the San Fernando dam. From the experimental results, they observed that $G_{\text{max}}$ decreased with an increase in the non-plastic $f_c$ up to 28% and then $G_{\text{max}}$ increased with $f_c$ of more than 28% (Figure 2.7). Chien & Oh (2002) conducted a series of resonant column test on Yun-Ling sand. They observed that $G_{\text{max}}$ increased with $f_c$ up to $f_c$ of 20% and then it decreased with further increase of $f_c$. Carraro et al. (2009) performed a series of tests with sands containing fines content using bender elements installed in the triaxial device. They reported that the effect of non-plastic $f_c$ on $G_{\text{max}}$ is more than plastic $f_c$ and there was a general trend of decreasing $G_{\text{max}}$ with non-plastic $f_c$. Wichtmann et al. (2015) performed a series of RC tests on a sand containing limited amount of fines content ($f_c < 20\%$). They found that $G_{\text{max}}$ decreases and damping ratio increases with an increase in fines content but their study was limited to the fines content of less than 20%.

Some efforts have been observed to capture the effects of non-plastic $f_c$ within the framework of Hardin’s relationship (Equation 2.7). Salgado et al. (2000) suggested that constant parameters, $A$ and $n$, in Hardin’s relationship might be affected by $f_c$. Iwasaki & Tatsuoka (1977) reported that parameter $A$ was affected by $f_c$ while other parameters remain constant. Therefore, they proposed a reduction factor for Hardin’s relationship.
However, the variation of reduction factor with $f_c$ are different for different host sands with fines (Figure 2.7). Wichtmann et al. (2015) correlated the fitting parameters of Hardin’s relationship to $f_c$.

In this research the resonant column test will be conducted on clean Hostun Sand as well as on clean Hostun Sand containing non-plastic fines, to find the effect of wide range of fines content on small and intermediate strain properties of granular materials. Furthermore, the relative performance of Hardin’s formulation (Equation 2.7) with the concept of the equivalent granular void ratio, $e^*$, will be discussed.

### 2.7 Sample preparation and fabric

Granular materials consist of discrete particles and voids between particles. Therefore, the response of such a discrete medium highly depends on the particle characteristics, e.g. stiffness, shape and angularity, contact characteristics and the arrangement of particles and contacts in assembly. Practically, soil structures, natural or man-made, are influenced by the generation procedure and gravitational field which induce an anisotropic fabric. During the deposition of soil particles, particles tend to be oriented in some preferred directions, initial inherent anisotropy in the soil structure, which eventually affects the
engineering properties of granular soils. The term fabric refers to the arrangement of particles, particle groups and pore spaces in a soil (Mitchell & Soga 2005). It is reasonable to assume that different sample preparation methods produce different soil fabrics and the macroscopic behaviour of granular material (i.e. stiffness and anisotropy) strongly depends on the fabric of granular assemblies (Oda 1972). Oda (1972) studied the effect of initial fabric, with different sample preparation, on the shear strength of two materials: 1- elongated particles and 2- spherical particles. They did triaxial tests on the prepared samples and observed that the effect of sample preparation on the peak stress ratio of the sample containing spherical particles was not obvious, however, a significant effect was observed for samples containing elongated particles which was due to the contact and particle orientation. Mahmood & Mitchell (1974) performed tests using direct shear test to show the effect of fabric on the shear strength of fine granular materials. They showed that grain arrangement and pore size distributions are different when samples are deposited in different ways. Tatsuoka et al. (1986) studied the effect of sample preparation on the cyclic undrained stress-strain behaviour of sand using triaxial and torsional shear tests. They prepared samples using four methods: air-pluviation, wet-tamping, wet-vibration and water-vibration. They reported that the cyclic undrained torsional shear strengths and cyclic undrained triaxial strengths were strongly affected by sample preparation methods. Vaid et al. (1999) performed a series of triaxial and direct shear tests on the Fraser River sand samples provided with different sample preparation. Samples were prepared by moist tamping, air pluviation, and water pluviation. From their experimental observations, they concluded that the sample preparation has a significant influence on the static undrained behavior of Fraser River sand. Wang & Mok (2008) performed a series of tests on Toyoura sand and rice particles using true triaxial device. From experimental observations, they concluded that the fabric-induced stiffness anisotropy increases with an increase in the aspect ratios of particles, which is the length ratio between the major and minor axes. Ezaoui & Di Benedetto (2009) studied the effect of sample preparation on the maximum moduli of Hostun sand using piezoelectric elements that were installed in the triaxial device. They prepared samples using three methods: i) pluviation method, ii) vibration method and iii) tamping method. They reported that the samples prepared by pluviation and vibration had a higher stiffness in the horizontal direction, whereas samples prepared by tamping induced higher stiffness in the vertical direction. Juneja & Raghunandan (2011) conducted a series of consolidated drained (CD) and consolidated undrained (CU) triaxial compression tests on sand samples prepared using pluviation and tamping techniques, under both dry and moist conditions. They reported that the stress-strain behaviour of samples prepared with various methods showed con-
2.8 Stress induced anisotropy

It is well-known that the stiffness of geo-materials depends on the state of stress in the soil elements. Hardin & Black (1966) studied the influence of the stress ratio on the maximum shear modulus of dry sand using a resonant column device. From experimental studies, Hardin & Black (1966) concluded that the shear modulus is independent of the deviatoric component of the initial stress state. Also, they concluded that Hardin’s relationship with sufficient accuracy can be used to predict the maximum shear modulus of soil elements subjected to anisotropic loading. Drnevich (1978) studied the influence of initial shear stress on the maximum shear modulus and damping of dry Ottawa Sand with the RC device and reported that the impact of initial shear stress on the maximum shear modulus is small. Also, they concluded that damping increases with the initial shear stress but the increase in damping is not significant. Tatsuoka et al. (1979) performed a series of cyclic torsional tests to evaluate the impact of static stress conditions on the small strain properties of Toyoura Sand using a torsional shear device. From their experimental results they concluded that the impact of extension loading on the maximum shear modulus is more remarkable than compression loading and they also concluded that the impact of the stress ratio and initial shear stress on the damping ratio is not significant. Yu & Richart (1984) studied the impact of the stress ratio on the maximum shear modulus of Brazil Sand and Ottawa Sand using a resonant column device. Yu & Richart (1984) concluded that $G_{max}$ can be affected by shear stress at a stress ratio of more than two. After these studies, Isenhower et al. (1987) modified Stokoe resonant column device to perform stress induced anisotropic tests. Santamarina & Cascante (1996) studied the effect of stress induced anisotropy on shear wave velocity in Barco Sand with the modified...
Stokoe resonant column device. However, their study was restricted to a stress ratio of less than two. Santamarina & Cascante (1996) reported that wave velocity will increase with an increase in the stress ratio but the effect of stress ratio on $\eta_{\text{min}}$ is not significant. All of the previous works to evaluate the effect of stress induced anisotropy on the maximum shear modulus were conducted on dense samples.

By reviewing the literature, systematic studies on the effect of stress induced anisotropy on the intermediate strain properties of granular material are rare, and we can refer to the study by Tatsuoka et al. (1979). They studied the effect of induced anisotropy on damping ratio and shear modulus on one stress path with a dense sample and reported that the impact of stress ratio on damping is not significant. However, their study was restricted to the stress path with a constant confining pressure on a dense sample. However, soil elements can be subjected to more complicated stress conditions in comparison with the stress conditions applied in the existing studies on intermediate strain properties up to now. Therefore, a major objective of this research work is to find the effect of stress induced anisotropy for different stress paths and density of sample on the small, and especially intermediate strain properties of sand. The resonant column device at Ruhr Universität Bochum was modified for performing such studies.
2.9 Summary

- The small strain properties of geomaterials can be explained by stress-strain hysteresis loops. Maximum shear modulus, $G_{\text{max}}$, shear modulus as a function of shear strain, $G(\gamma)$ and damping ratio, $\eta$, are the main parameters which control the response of the soil elements subjected to vibration. They are also the key parameters in soil-structure interaction problems with small deformations under rapid and repetitive loads.

- The effect of $f_c$ on $G_{\text{max}}$ and $\eta(\gamma)$ must be evaluated with a systematic increase of $f_c$. Application of the equivalent granular void ratio to predict the small strain stiffness of granular material must be studied. The data sets collected from the literature were limited to the maximum 20% $f_c$. Thus, a suitable approach for capturing the effect of a wide range of $f_c$ on $G_{\text{max}}$ and $\eta(\gamma)$, beyond threshold fines content, is still a topic of research interest.

- Studies on the effect of stress induced anisotropy on the small and intermediate strain properties can be divided into two main groups: 1-studies on the small strain properties ($G_{\text{max}}$), and 2-studies on intermediate strain properties ($G(\gamma)$ and $\eta(\gamma)$). Concerning 1: As is apparent from the literature review, numerous experimental studies have been conducted by piezoelectric elements for evaluating the effect of induced anisotropy on $G_{\text{max}}$ (small strain properties) of granular materials, but some of them (e.g. Hardin & Black 1966, Tatsuoka et al. 1979, Yu & Richart 1984 and Santamarina & Cascante 1996) used the resonant column device and they concluded that at a stress ratio of less than two, the effect of stress induced anisotropy on $G_{\text{max}}$ is not significant but these studies were restricted to dense samples. Concerning 2: From the literature review we can see that studies on the effect of induced anisotropy on the intermediate strain properties of granular material are rare, and we can refer to the study by Tatsuoka et al. (1979). They studied the effect of induced anisotropy on damping ratio and shear modulus for one stress path (confining pressure constant and vertical pressure variable) on dense samples and reported that the impact of the stress ratio on damping is not significant. However, soil elements can be subjected to more complicated stress conditions in comparison to the stress conditions applied in the studies conducted on intermediate strain properties up to now. Therefore, additional studies are essential to assess the effect of stress induced anisotropy for different stress paths on the small, and especially
intermediate strain properties of sand. Furthermore, the empirical relations must be developed to predict $G(\gamma)$ of soil samples subjected to stress induced anisotropy for different stress paths.
3 Material and experimental program

3.1 Introduction

The adopted experimental program to assess the influence of fines content and stress induced anisotropy on the small and intermediate strain properties of granular materials will be presented in this chapter. This chapter has three main objectives: Firstly, the physical and mechanical properties of adopted materials are discussed. Hostun sand and glass beads are introduced as host and main materials used in this study. Also, two mixtures, Hostun Sand containing quartz powder and glass beads containing fine glass beads, are introduced as a mixture to study the effect of fines content. Secondly, the apparatus used for the experimental study is introduced. Before performing tests, calibration and validation of the apparatus are essential. Therefore, the numerical, theoretical and experimental procedures for the calibration and validation of the apparatus are discussed. The effect of vertical load on the dynamic behaviour of the system (resonant column device) will be examined with the numerical modeling of the device. Finally, the experimental procedure for performing the isotropic and stress induced anisotropic tests will be presented.

3.2 Materials

Two types of materials were used in this research: 1) Hostun Sand; and 2) glass beads. Both of the materials were adopted as host materials for studying the effect of fines and stress induced anisotropy on small and intermediate strain properties. The physical properties of these materials are presented in the following sections.
3.2.1 Hostun Sand

The main experimental program for this study was conducted on Hostun Sand which has been used in the previous studies (Flavigny et al. 1990, Schanz & Vermeer 1996 and Sadek 2006). The color of Hostun Sand ranges between gray-white and rosy-beige, while its chemical components consist of a large amount of siliceous, $SiO_2 > 98\%$, (Amat 2007). Hostun Sand is a quartz sand with grain sizes ranging from 0.1 mm to 1.0 mm. Figure 3.1a shows the microscopic image of particles. The grain shape varies from angular to sub-angular. According to the USCS classification, the material is a poorly-graded medium sand SP (Lins 2009). The values of the maximum and minimum void ratios, $e_{\text{max}}$ and $e_{\text{min}}$, of Hostun Sand were measured using DIN 16126 and the results are summarized in Table 3.1.

A particle size distribution analysis was carried out using DIN 18123 standard. Figure 3.1b shows the particle size distribution of Hostun Sand.

The stress-strain behaviour of Hostun Sand was essential for the discussion of the observed experimental results using the resonant column device. Hence, triaxial tests were conducted at the confining pressure of 200 kPa, $\sigma_3=200$ kPa, on dense ($D_r = 90\%$) and loose ($D_r = 35\%$) samples. The initial conditions in triaxial tests were the same as the conditions for the samples in resonant column tests. The stress-strain curve and volumetric changes of dense and loose Hostun Sand are presented in Figure 3.2a and Figure 3.2b.

![Figure 3.1: Particle characteristics of Hostun Sand: (a) shape of particles; (b) grain size distribution](image)
3.2 Materials

Figure 3.2: Triaxial behaviour of dry Hostun Sand at the confining pressure of 200 kPa, $\sigma_3=200$ kPa,: (a) vertical stress versus vertical strain for loose and dense samples; (b) volumetric strain versus vertical strain for loose and dense samples

These figures show the well-known behaviour of dense and loose sand by increasing the vertical strain. Figure 3.2a shows the samples reach the critical state zone at vertical strain of more than 15%. Also, the results in Figure 3.2b show that the dense sample dilated at the vertical strain by more than 1.5% and the loose sample compacted with an increase in the vertical strain.

3.2.2 Glass beads

Spherical glass particles with a diameter of 1.25 mm were used as the second host material for studying the effect of fine particles and stress induced anisotropy on small and intermediate strain properties. The SEM photographs, scanning electron microscope, of glass beads are shown in Figure 3.3. The stress-strain behaviour of the adopted glass beads were essential for the calibration and validation of the prepared model in DEM simulation and the stress-strain results were also essential for interpretation of the test results from the resonant column test. Discrete element simulations and resonant column tests were conducted on the dense glass bead samples. Therefore, triaxial tests were conducted on the dense glass bead packing ($Dr = 88\%$) at the confining pressure of 200 kPa, $\sigma_3=200$ kPa, for calibration of the DEM model. Figure 3.4 shows the behaviour of the dense glass bead packing obtained from the triaxial test. Figure 3.4a demonstrates that the vertical stress in the dense sample reached the maximum value at a vertical strain of less than
2% and dilation occurred at a vertical strain of more than 0.6% (Figure 3.4b). The other physical properties of the adopted glass particles are summarized in Table 3.2.

Figure 3.3: Particle characteristics of glass beads: (a) coarse particles, D=1.25 mm; (b) fine particles, d=0.125 mm

Figure 3.4: Triaxial behaviour of dry dense glass beads ($D_r = 88\%$) at the confining pressure of 200 kPa, $\sigma_3=200$ kPa,: (a) vertical stress versus vertical strain; (b) volumetric strain versus vertical strain
3.3 Mixtures

Two types of mixture were used in this study: 1) Hostun Sand mixed with fine quartz powder 2) glass beads mixed with glass fines particles which are discussed in this section.

3.3.1 Hostun Sand and quartz powder

Hostun Sand was washed to separate the particles with a diameter of less than 0.075 mm, so-called clean Hostun Sand. Clean Hostun Sand was mixed with 0, 5, 10, 20, 30 and 40 gravimetric percentages of non-plastic fines. The adopted non-plastic fines were a commercial quartz powder (M500), composed of 99% SiO$_2$, from the Euro-quartz company. Before performing tests with the prepared mixtures, the maximum and minimum void ratio of the mixtures must be determined. There is no applicable ASTM procedure to determine the maximum and minimum void ratios of soils with 15% or more fines content (Tao et al. 2004) which is due to the segregation of particles during pluviation. However, the ASTM standard has been used to determine the maximum and minimum void ratio for coarse materials containing more than 15% fines in previous works (e.g. Tao et al. 2004). In this research, the procedure adopted to determine the variation of $e_{\text{max}}$ and $e_{\text{min}}$ with the systematic increase of fines content was the same as the procedure presented in DIN 18126. Regarding this standard, the value of $e_{\text{max}}$ was determined by placing the standard funnel at the bottom of the standard mold. Then, the funnel was raised slowly without dropping the particles which reduces particles segregation. To determine the value of $e_{\text{min}}$ the sample, from the $e_{\text{max}}$ test, was devided into five equal layers. After placing each layer inside the mold, the mold was tapped uniformly with a standard hammer. The variation of $e_{\text{max}}$ and $e_{\text{min}}$ for Hostun Sand with different percentages of $f_c$ and the other physical properties of the mixtures are summarized in Table 3.1.

Table 3.1: The physical properties of Hostun Sand-quartz powder mixtures

<table>
<thead>
<tr>
<th>Soil</th>
<th>$e_{\text{max}}$</th>
<th>$e_{\text{min}}$</th>
<th>$\gamma_{\text{dmin}}$</th>
<th>$\gamma_{\text{dmax}}$</th>
<th>$G_s$</th>
<th>$D_{50}$</th>
<th>$D_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clean Hostun sand</td>
<td>1.023</td>
<td>0.671</td>
<td>1.310</td>
<td>1.586</td>
<td>2.65</td>
<td>0.385</td>
<td>0.211</td>
</tr>
<tr>
<td>CHS+5%$f_c$</td>
<td>1.097</td>
<td>0.642</td>
<td>1.264</td>
<td>1.614</td>
<td>2.65</td>
<td>0.380</td>
<td>0.172</td>
</tr>
<tr>
<td>CHS+10%$f_c$</td>
<td>1.219</td>
<td>0.635</td>
<td>1.194</td>
<td>1.624</td>
<td>2.65</td>
<td>0.379</td>
<td>0.082</td>
</tr>
<tr>
<td>CHS+20%$f_c$</td>
<td>1.501</td>
<td>0.624</td>
<td>1.072</td>
<td>1.630</td>
<td>2.65</td>
<td>0.337</td>
<td>0.004</td>
</tr>
<tr>
<td>CHS+30%$f_c$</td>
<td>1.739</td>
<td>0.682</td>
<td>0.968</td>
<td>1.575</td>
<td>2.65</td>
<td>0.324</td>
<td>0.003</td>
</tr>
<tr>
<td>CHS+40%$f_c$</td>
<td>1.925</td>
<td>0.791</td>
<td>0.906</td>
<td>1.479</td>
<td>2.65</td>
<td>0.266</td>
<td>0.002</td>
</tr>
</tbody>
</table>
Table 3.2: The physical properties of glass bead-glass bead mixtures

<table>
<thead>
<tr>
<th>f_c [%]</th>
<th>e_{max} [-]</th>
<th>e_{min} [-]</th>
<th>( \gamma_{d_{min}} ) [g/cm^3]</th>
<th>( \gamma_{d_{max}} ) [g/cm^3]</th>
<th>G_s [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.6851</td>
<td>0.5616</td>
<td>1.4836</td>
<td>1.5908</td>
<td>2.50</td>
</tr>
<tr>
<td>10</td>
<td>0.5706</td>
<td>0.4356</td>
<td>1.5817</td>
<td>1.7414</td>
<td>2.50</td>
</tr>
<tr>
<td>20</td>
<td>0.47</td>
<td>0.3399</td>
<td>1.667</td>
<td>1.8659</td>
<td>2.50</td>
</tr>
<tr>
<td>30</td>
<td>0.3849</td>
<td>0.2698</td>
<td>1.7423</td>
<td>1.9369</td>
<td>2.50</td>
</tr>
<tr>
<td>40</td>
<td>0.3973</td>
<td>0.2767</td>
<td>1.734</td>
<td>1.9336</td>
<td>2.50</td>
</tr>
<tr>
<td>50</td>
<td>0.4225</td>
<td>0.3064</td>
<td>1.7224</td>
<td>1.9284</td>
<td>2.50</td>
</tr>
</tbody>
</table>

The test results show the \( e_{min} \) decreased up to fines content of 20% and then increased with further increases of \( f_c \). The \( f_c \), that is the boundary between the “fines-in-sand” and "sand-in-fines" micro-structure, is called the threshold fines content, \( f_{thr} \) (Rahman & Lo 2008). Regarding the \( e_{min} \) analysis, the value of \( f_{thr} \) is the fines content that \( e_{min} \) starts to increase with further increases in fines content (Naeini & Baziar 2004 and Zuo & Baudet 2015). From Table 3.1, the value \( f_{thr} \) should be in between 20% and 30% for Hostun Sand mixed with quartz powder which was assumed to be 25% for this study.

### 3.3.2 Glass bead mixtures

Coarse glass beads (D=1.25 mm) were mixed with 0, 10, 20, 30, 40 and 50 gravimetric percentages of spherical fine glass beads (d=0.125 mm, Figure 3.3b). The values of \( e_{max} \) and \( e_{min} \) was determined for all of the mixtures, the procedure was the same as the adopted procedure for the Hostun Sand mixtures. The variations of \( e_{max} \) and \( e_{min} \) for the glass bead mixtures are summarized in Table 3.2. The results show that the value of \( e_{min} \) decreased up to 31% \( f_c \) and then increased with a further increase of \( f_c \).

### 3.4 Sample preparation

Dry funnel deposition and the air pluviation method are two common methods for sample preparation. In the air pluviation method, dry sands are pluviated through a diffuser from a constant height and thus may not be suitable for sand with more than 15% \( f_c \), due to particle segregation. In the dry deposition method, the specimens were prepared
by placing the funnel at the bottom of the mold. Then the material were placed inside the funnel and the funnel was raised slowly which reduced particle segregation. The mold was tapped in a symmetrical pattern to achieve a dense specimen. This technique was commonly used for testing silty sands by Ishihara (1993), Lade & Yamamuro (1997), Yamamuro & Lade (1997) and Monkul & Yamamuro (2011). Therefore, the dry deposition method, which gives the most uniform specimen compared to pluviation for mixtures, was used for the sample preparation in this study.

3.5 Apparatus

3.5.1 Description of equipment

Resonant column and piezoelectric elements are two common laboratory devices for measuring the small and intermediate strain properties of geo-materials. The piezoelectric elements are mounted in the different laboratory devices to measure the maximum shear modulus at a low amplitude of vibration (e.g. Jovicic & Coop 1988, Kuwano et al. 2000, Kuwano & Jardine 2004 and Wang & Mok 2008). These elements are also applicable when determining the modulus reduction curves and damping ratio of soils (e.g. Karl et al. 2008, Choo et al. 2013) although it is not a standard procedure.

Besides the piezoelectric elements, the resonant column device is one of the standard methods (ASTM D-4015) for measuring small and intermediate strain properties. The resonant column technique is based on the oscillation of the cylindrical sample on one of its vibration modes, flexural mode (Cascante et al. 1998), normal mode (Saxena & Reddy 1989) and torsional mode (a common type of resonant column), to determine the resonant frequency of a sample.

The torsional resonant column device is the common type of resonant column. The torsional resonant column device was initially adopted by Iida (1937), a Japanese researcher, to evaluate the influence of water content on wave velocity (Figure 3.5a). This device was developed in the last few decades, e.g. Hardin & Black (1966), Hardin & Drnevich (1972b), Hardin & Drnevich (1972a), Drnevich (1978) and Stokoe et al. (1999a), for studying the small strain properties of soils (Figure 3.5b). The torsional resonant column device is divided into two groups, free-free and fix-free resonant column devices which are appropriate for determining the mechanical properties of soils with a wide range of strain, from a very small strain to medium strain (Figure 1.3).
In free-free resonant column device, the actuator is mounted on the top or bottom of the sample whereas the other end is free in rotation. In the fix-free resonant column device, one side of the sample is constrained against rotation and the other side is free in rotation (Drnevich resonant column and Stokoe resonant column). The resonant column device at Ruhr-Universität Bochum (Figure 3.6) was designed so that the polar mass moment of inertia in bottom part ($J_0$) is about 17.2 times the polar mass moment of inertia in the top part ($J_L$), ($J_0/J_L = 17.2$). Thus, this device can be categorized in a free-free resonant column device group and relationships must be developed based on the free-free boundary conditions. Regarding the boundary conditions of the Bochum resonant column device, the Equation 3.1 and Equation 3.2 are applicable for determining the maximum shear modulus of materials (Richart et al. 1970 and Wichtmann et al. 2001).

$$\tan(a) = \frac{J^2}{J_0 J_L} \frac{\tan(a)}{a} = \frac{J}{J_0} + \frac{J}{J_L}$$  \hspace{1cm} (3.1)

$$G = \left(\frac{2\pi f_r}{a}\right)^2 \rho$$  \hspace{1cm} (3.2)

where, $a$ is $\omega L/v_s$, $\omega$ is the rotational frequency, $v_s$ is the wave velocity, $J_0$ and $J_L$ are the polar mass moment of inertia for the bottom and top parts of the resonant column, $J$ is the polar mass moment of inertia for the sample, $\rho$ is the density of the sample, $f_r$, is the resonant frequency and $L$ is the height of the sample. By assuming the infinite value for $J_0$ in Equation 3.1, the relationship will be the same as the proposed relationship for the fix-free resonant column device. The top part of the Bochum resonant column device (Figure 3.7a) includes two mini-shakers for applying a rotational excitation at the top.
of the sample. Two transducers (accelerometers) were mounted on the mini-shakers and the actuator were used for monitoring the signals (Pkk\textsubscript{1} and Pkk\textsubscript{2}, Figure 3.7a) using oscilloscope device. Phase difference between the signals received from the mini-shakers and displacement transducers (Pkk\textsubscript{1} and Pkk\textsubscript{2}) must be $\pi/2$ at the resonant frequency.

Figure 3.6: The torsional resonant column device at Ruhr Universität Bochum

Figure 3.7: The Bochum resonant column device: (a) the actuator or top part of the Bochum resonant column device; (b) top view of assembled device
According to Equation 3.1 and Equation 3.2, three constant values \((J_L, J_0 \text{ and } J)\) are key parameters for determining \(G_{\text{max}}\) with the resonant column device. To increase the accuracy of the results, these values must be determined accurately.

The bottom part of the Bochum resonant column device and the sample have a cylindrical shape with a given density and dimensions, thus the polar mass moment of inertia \((J_0 \text{ and } J)\) of these parts can be determined using geometric relations. The top part of the Bochum resonant column device (Figure 3.7a) has a complex shape, including electrical equipment, cables and holes, thus the polar mass moment of inertia for this part \((J_L)\) can not be determined accurately with geometric methods. Hence, experimental methods must be used to determine the value of \(J_L\). By a simple experimental procedure the value of \(J_L\) can be calculated. With this method, the shear modulus of the aluminum sample \((G)\) was assumed to be a value between 24 and 27 GPa, and then, the resonant frequency of the aluminum sample is measured using the resonant column device. Hence, the parameter \(a\) can be determined by substituting the shear modulus and the measured resonant frequency in Equation 3.2. The mass moment of inertia for the top part \((J_L)\) can be calculated by substituting the determined values for \(a\), \(J\) and \(J_0\) in Equation 3.1. In this study, the stiffness of aluminum \((G)\) was an unknown value, thus \(G\) was assumed to be 25 GPa. Therefore, this experimental method includes an error because of the stiffness of the aluminum sample. Another experimental method, which is independent of the shear modulus \((G)\) of the aluminum sample, was used to determine the value of \(J_L\) with a high degree of accuracy. The experimental method developed by Tatsuoka & Silver (1980) for the fix-free resonant column device was employed for determining the value of \(J_L\) in the free-free resonant column device. In this experimental method, an extra mass with given density and geometry was used to determine the value of \(J_L\). The extra mass was placed on the top of the actuator and the resonant column tests were conducted for both of the conditions (with and without extra mass). By rewriting Equation 3.1 and Equation 3.2, three unknown values: \(a\), \(b\) and \(J_L\) were defined.

\[
\begin{align*}
    a \tan(a) - \frac{J^2}{J_0 J_L} \tan(a) a &= \frac{J}{J_0} + \frac{J}{J_L} \quad (3.3) \\
    b \tan(b) - \frac{J^2}{J_0 (J_L + J_{\text{mass}})} \tan(b) b &= \frac{J}{J_0} + \frac{J}{(J_L + J_{\text{mass}})} \quad (3.4) \\
    \frac{a}{b} &= \frac{f_{r1}}{f_{r2}} \quad (3.5)
\end{align*}
\]

where, \(f_{r1}\) is the resonant frequency of the aluminum sample without the extra mass and \(f_{r2}\) is the resonant frequency of the aluminum sample with the extra mass. According to the dimension of the sample (2 cm in diameter and 30 cm in height), the mass moment
of inertia of the sample \((J)\) was about \(1.2723E - 5 \text{ kg.m}^2\). Also, the value of \(J_0\) could be determined by the geometric method and was \(1.171415 \text{ kg.m}^2\). The resonant frequency of the aluminum sample without the extra mass was determined by a resonant column test which was 22.82 Hz \((f_{r1})\). The extra mass with a given mass moment of inertia \((J_{mass}=0.0148 \text{ kg.m}^2\) was placed on the top of the actuator and the resonant frequency of the aluminum sample was measured \((f_{r2})\) which was 20.71 Hz. By substituting the measured resonant frequencies in Equation 3.5, the value of \(b\) will be equal to \(ka = 0.9075a\).

Therefore, Equation 3.3, Equation 3.4 and Equation 3.5 were reduced to Equation 3.6 and Equation 3.7 with two unknown parameters \((a\) and \(J_L\)).

\[
F(a, J_L) = a \tan(a) - \frac{J^2}{J_0 J_L} \tan(a) \frac{a}{J_0} - \frac{J}{J_0} - \frac{J}{J_L} = 0 \tag{3.6}
\]

\[
G(a, J_L) = ka \tan(ka) - \frac{J^2}{J_0(J_L + J_{mass})} \tan(ka) \frac{ka}{J_0} - \frac{J}{J_0} - \frac{J}{(J_L + J_{mass})} = 0 \tag{3.7}
\]

where, \(k\), \(J\), \(J_0\) and \(J_{mass}\) are equal to 0.9075, 1.2723E - 5 \text{ kg.m}^2, 1.171415 \text{ kg.m}^2\ and 0.0148 \text{ kg.m}^2\ respectively. From the performed calculations, \(J_L\) is equal to 0.0647 \text{ kg.m}^2\.

The value of \(J_L\) was also calculated with the geometric method and its value with geometric relations was about 0.0659 \text{ kg.m}^2\ but the \(J_L\) from the experimental method was used in all of the tests with the resonant column device on the soil samples.

### 3.5.2 Mode of vibration

Two types of rotational modes may be achieved during the test with the free-free resonant column: 1- actual rotational mode; and 2- spurious rotational mode (defined by Avramidis & Saxena 1990). The actual rotational mode is achieved when there is an phase difference between the top and bottom of the sample during the rotational excitation of the sample. The other one, spurious or fake mode, is achieved when the top and bottom of the sample have the same rotational phase during the RC test to avoid this mode of vibration mass moment of inertia of free side \((J_0\) in Bochum RC) must be increased or samples with less stiffness must be used. The rotational mode of vibration must be distinguished during isotropic and anisotropic tests with the resonant column device. Two accelerometers were installed on the rotatable bottom part of the resonant column device, Acc.2, (Figure 3.8a) to determine the type of rotational mode, actual or spurious. The received signals from the accelerometers, Acc.1 and Acc.2, were monitored and compared with each other during the resonant column test.
Due to the shape of actuator, additional mode of vibration could be achieved during tests with the Bochum resonant column device which is called flexural or bending mode of vibration. This mode of vibration was monitored using Acc.3 which was mounted in one side of the actuator (Figure 3.8a). The received signals from Acc.3 showed that the amplitude of signals were not significant in comparison with the applied torsional vibration. This means that the flexural mode of vibration can be ignored during tests using the
Bochum resonant column device.

Figure 3.8b shows an example of the signals received from Acc.1 and Acc.2. Figure 3.8b shows an example of the applied signals at the top of the sample in comparison with the received signals from the bottom of the sample. As can be seen in this figure, when the peak value of the signals applied to the top of the sample (Acc.1) was 0.97v, the peak amplitude of the signals received from the bottom was about 0.38v which means the bottom of the sample is free in rotation. Furthermore, Figure 3.8b shows a $\pi$ phase difference between the received signals which is due to the direction of the installed accelerometers at the top and bottom of the RC (both of them are in the same direction).

The obtained results show that the bottom and top of the sample rotate at different phases. The resonant frequency will be determined for the actual rotational mode of excitation. This frequency is achieved when the phase difference between $P_{kk1}$ and $P_{kk2}$ signals is $\pi/2$ (Figure 3.9b). For ease of determining the resonant frequency, $P_{kk2}$ was drawn versus $P_{kk1}$ (Figure 3.9a) on the oscilloscope device. Then the frequency of the applied signal was changed to get the resonant frequency; at the resonant frequency the generated ellipse shape must be vertical (Figure 3.9b).

### 3.5.3 Modification and calibration of apparatus

#### 3.5.3.1 Loading system

The actuator of the Bochum resonant column device is the cubic part which includes the electrical equipment (Figure 3.6). To apply additional vertical stress inside of the sample, the actuator was loaded in a vertical direction using a double acting pressure cylinder added by APS Wille Geotechnik GmbH, Goettingen, (Figure 3.10). The load of the cylinder was transferred with a loading bar, a hardened steel tip, to the hardened steel plate which was mounted at the central axis of the actuator and sample (Figure 3.10). Firstly, the relationship between pressure $p$ and the generated force in the loading bar, $F$, must be determined. The calibrated load cell was used to determine the value of $F$ with systematic increase of $p$ in double acting pressure cylinder.

Figure 3.11 shows the relationship between the applied pressure, $p$, and $F$ in the loading bar. In this calibration, $p_0$ was assumed to be constant and equal to 50 kpa. The value of $p_0$ was applied to improve the motion of the loading bar during the unloading process. Then, the effect of cell pressure ($\sigma_3$) on the value of $F$ must be calculated (Figure 3.10b). As can be seen in Figure 3.10b, cell pressure has both a negative and positive effect on the magnitude of $F$ in the loading bar which must be assessed.
Figure 3.10: Instrumentation for applying vertical load: (a) schematic sketch of double acting pressure device and loading bar; (b) the effect of cell pressure on loading bar

Figure 3.11: Calibration of double acting pressure cylinder, $p_0=50$ kPa
3.5 Apparatus

The effect of cell pressure on the vertical load, \( F \), was calculated by writing the equilibrium relationship in a vertical direction (Equation 3.8).

\[
F_v = F + A_1(\sigma_3) - A_2(\sigma_3)
\]  

(3.8)

Where, \( A_1 \) and \( A_2 \) are equal to 2.155 cm\(^2\) and 5.02 cm\(^2\) respectively. From the equilibrium relation, the value of \( F_v \) was a function of cell pressure.

Finally, the influence of the loading equipment on the dynamic behaviour of the system (bottom mass, sample and actuator) and its interaction with the actuator must be assessed. Therefore, the finite element method, ABAQUS software, and experimental methods, using aluminum samples, were used for this assessment.

3.5.3.2 The effect of the loading bar

Damping ratio in aluminum specimens is approximately zero (Zemanek & Rudnick 1961). Therefore, the resonant frequency of aluminum samples is approximately equal to the natural frequency which allows modeling of the resonant column device with the aluminum sample in ABAQUS software. This type of analysis was helpful to evaluate the effect of loading and interaction between the loading bar and the actuator on the natural frequency of the aluminum sample and the dynamic behaviour of system. A numerical model was developed to analyze the behavior of the RC device without a vertical loading system. After validation of the model, the numerical model was extended to analyze the influence of the vertical loading system on the behaviour of the RC device. In the numerical model, the RC device was idealized into three main parts: actuator, sample and bottom part. The geometries of these parts were modeled the same as the real device (Figure 3.12). All of the connections, e.g. connections between the actuator and the top of the sample, and the bottom part of the device and sample, were assumed to be rigid connections. Material properties of the modeled parts were the same as the real device in the laboratory (the actuator and the sample was made of aluminum and the rotatable bottom part was made of steel). The top part of the modeled device was free for displacement or rotation but the bottom part of the system was constrained against displacement and it was free in rotation only, the same as the boundary conditions of the device in the laboratory.

For validation of the model, the frequency analyses were conducted to determine the natural frequency of the aluminum samples with a diameter of 2, 4 and 6 cm and 30 cm height. Two types of rotational frequencies were achieved from the numerical model, actual and spurious frequency (Figure 3.12a and Figure 3.12b). As is apparent from Figure 3.12a, the top and bottom of the model have the same colour. This means the
top and bottom of the model have the same rotational phase. In this case the rotational frequency is the so-called spurious frequency. However, Figure 3.12a shows the actual mode of vibration and this frequency is used to determine the shear stiffness. Therefore, the obtained actual frequencies from the numerical model for aluminum samples were extracted and presented in Tables 3.3 and 3.4. Table 3.3 shows the measured frequencies of aluminum samples without vertical loading which was calculated with different methods. The results confirm a good agreement between frequencies calculated by the numerical model and measured values using the resonant column device (Table 3.3).

![Image of numerical models](image)

Figure 3.12: The obtained rotational frequencies for aluminum sample No.4 from numerical analysis: (a) actual mode of vibration; (b) spurious mode of vibration

<table>
<thead>
<tr>
<th>Specimen</th>
<th>RC test</th>
<th>Theory Equations 3.1 and 3.2</th>
<th>Modeled in ABAQUS</th>
</tr>
</thead>
<tbody>
<tr>
<td>t [cm]</td>
<td>fr [Hz]</td>
<td>fr [Hz]</td>
<td>fr [Hz]</td>
</tr>
<tr>
<td>2</td>
<td>22.82</td>
<td>23.43</td>
<td>23.371</td>
</tr>
<tr>
<td>4</td>
<td>92.41</td>
<td>93.45</td>
<td>91.707</td>
</tr>
<tr>
<td>6</td>
<td>182.95</td>
<td>209.86</td>
<td>198.523</td>
</tr>
</tbody>
</table>
3.5 Apparatus

Table 3.4: The measured resonant frequency for the aluminum sample using the numerical method, with vertical load

<table>
<thead>
<tr>
<th>Specimen t [cm]</th>
<th>Diameter of contact area s [cm]</th>
<th>Vertical load $F_v$ [N]</th>
<th>Modeled in ABAQUS $f_r$ [Hz]</th>
<th>Effect of loading bar $\Delta 1$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
<td>133.45</td>
<td>139.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>0</td>
<td>91.791</td>
<td>92.078</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0</td>
<td>91.713</td>
<td>91.754</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

After validation of the numerical model, the effect of the vertical load on the dynamic behaviour of system was considered. For approaching to this aim, additional modification was applied to the prepared numerical model to evaluate the effect of the loading bar. A rigid cylindrical element was defined as a loading bar in the FEM model. The loading bar, in FEM model, was constrained in all of the directions but it was free in the vertical displacement. One step was defined before the frequency analysis step to develop the effect of the vertical load on the obtained frequency from the modeled device. The mechanical changes like loading and friction interaction between the loading bar and the actuator were defined in this step of analysis and their effects were observed during frequency analysis in the second step. The effect of two main variables on the response of the system during loading were studied: 1- the effect of the diameter of connection, $s$, between the loading bar and the actuator ($s$ was equal to 1, 0.2 and 0.1 cm); and 2- the effect of the vertical load, $F_v$, which was equal to 0 and 4000 N. In all of the analyses the fiction coefficient between the loading bar and the actuator was assigned as a smooth steel-steel friction coefficient (assumed 0.3). The results of the FEM analysis are summarized in Table 3.4.

For ease of discussion, the frequency of aluminum sample ($t = 4cm$) without the vertical load (Table 3.3) was assumed to be a reference value. Table 3.4 shows that the frequency of the aluminum sample increases by 45.52% with increase in the diameter of connection, $s$, to 1cm which is due to the friction between the loading bar and actuator. By increasing the vertical load from 0 to 4000 N, the frequency increased by 51.82% which is due to the increase in the friction and, consequently, the generated resistance moment between the loading bar and the actuator. Table 3.4 shows that by decreasing the connection area from 1 to 0.1 cm the natural frequency of the aluminum sample increases by 0.045% in comparison with the reference value and by increasing the vertical load from 0 to 4000 N,
the natural frequency increases by 0.05%. This means the effect of the loading bar and the vertical load on the response of the device decreases with an decrease in the resistance moment in the connection area. The variations of $\Delta_1$ in Table 3.4 show that the effect of the vertical load on the dynamic behaviour of the device decreases with a decrease in the connection area ($s$) and generated resistance moment. Therefore, the rotational frequency of the aluminum sample was affected by the generated resistance moment at contact point which was due to the frictional interaction between the loading bar and actuator. This resistance moment can be reduced by minimizing the contact area and decreasing the friction between the loading bar and actuator. According to this argument, the effect of point loading (loaded on central axis of device) on the rotational mechanism of device can be neglected. For decrease the friction in the connection area, the diameter of the connection area between the loading bar and the top cap of the resonant column device was decreased to less than 1 mm and also a smooth steel plate with a high stiffness and the minimum friction coefficient was added to the top of the actuator.

3.5.4 Validation of modified apparatus

3.5.4.1 Validation with the aluminum sample

Aluminum samples with diameters of 2, 4 and 6 cm and 30 cm in height (defined as samples No.2, No.4 and No.6) were used to control the compliance (defined by Clayton et al. 2009) of the resonant column device. Three methods were used to determine the resonant frequency of the aluminum samples. In method one, the aluminum samples were mounted in the resonant column device and the resonant frequency of the aluminum samples and, consequently, the stiffness of the samples were determined. In the second method, so-called theoretical method, the stiffness of aluminum samples was assumed 25 GPa and the resonant frequency of the samples was determined using Equation 3.1 and Equation 3.2. In the third method, the finite element model was used to determine the frequency of the samples (as discussed above). The calculated resonant frequencies for the aluminum samples with these methods are summarized in Table 3.3. As can be seen in Table 3.3, error of the device for sample No.2, in comparison with the theoretical method, is more than error of the device for aluminum sample No.4. This difference is due to the compliance of the sample, because the stem of sample No.2 in comparison with its length is so thin, therefore rotational signals may be affected by flexural signals during tests with this sample. As can be seen in Table 3.3, a significant error was obtained for sample No.6, which was due to the type of rotational mode. Sample No.6 was too stiff and thus the top
and bottom of the sample rotated in the same phase. In this situation, a spurious mode was measured instead of the actual mode.

Aluminum sample No.4 was chosen for the calibration and validation of the device for induced anisotropic tests. Aluminum sample No.4 was installed in the resonant column device and vertical load was increased from 0 to 3000 N. A small increment (0.88%) was observed in the obtained frequency for the aluminum sample (Table 3.5). The observed increment may be due to the compliance of the aluminum sample and connections.

### 3.5.4.2 Validation with soil sample

Piezoelectric elements (shear plates) were installed in the top and bottom caps of the modified resonant column device (Figure 3.6). A series of resonant column and piezoelectric element tests were conducted on dry samples subjected to isotropic and stress induced anisotropic loading.

![Figure 3.13: The effect of void ratio and confining pressure on maximum shear modulus](image-url)
Figure 3.13 shows a decrease in maximum shear modulus with an increase in void ratio and a decrease in confining pressure for dry Hostun Sand subjected to isotropic confining pressure. The increase in stiffness with an increase in the confining pressure is due to the increase in the normal contact forces at contact points (will be discussed in Chapter 5).

Four stress paths were defined (Figure 3.14a) to perform the stress induced anisotropic tests on dry Hostun Sand. In this experimental program shear plates (SP) were mounted in the top and bottom caps of the resonant column device and stress induced anisotropic tests were conducted with both of the devices simultaneously. The measured maximum shear modulus with both of the methods are compared with each other in this section.

Figure 3.14: The experimental results using the resonant column device and shear plate elements: (a) stress paths; (b) stress path: I, Dr=35%−90%; (c) stress path: II, Dr=50%−90%; (d) stress path: III, Dr=70%−90%
Two statistical measures of Root-Mean-Square-Deviation (RMSD) and $R^2$ were used to estimate the scatter of measured results by the resonant column in comparison with SP. For stress path I, vertical pressure was increased and confining pressure was kept constant at 200 kPa during RC tests. The measured maximum shear modulus ($G_{\text{max}}$) with RC and SP tests for this stress path is presented in Figure 3.14b. This figure confirms a good agreement between the obtained results with the modified resonant column device and bender elements at this stress path. For stress path II, $p' = \frac{\sigma_1 + 2\sigma_3}{3}$ was kept constant at 200 kPa during RC tests and $q = \sigma_1 - \sigma_3$ was increased. The values of $G_{\text{max}}$ for dry Hostun Sand with relative density of 70%–90% for this stress path are presented in Figure 3.14c. Figure 3.14c shows a good agreement between the obtained results with the modified resonant column device and shear plate elements at this stress path. For the third stress path (stress paths III), the constant stress ratio was used to compare and contrast the obtained experimental results ($K = \frac{\sigma_3}{\sigma_1} = 0.5$). As is clear in Figure 3.14d, the obtained results from the modified resonant column device are consistent with the obtained results from shear plate elements.

Sadek et al. (2007) conducted a comprehensive series of stress induced anisotropic tests on Hostun Sand with a relative density of 65% using cubical cell apparatus designed at Bristol. The results obtained by Sadek (2006) and the results from the modified resonant column device on the same stress path are presented in Figure 3.15.

![Figure 3.15: Maximum shear modulus for Hostun Sand, obtained by the cubical cell apparatus, Sadek (2006) and the modified resonant column](image-url)
As can be seen in this figure, there is a good agreement between the results from the modified resonant column device and the results presented by Sadek (2006), although the device and the test methods are quite different in both of the experimental studies.

### 3.5.5 Calibration for damping

The objective of this section is to validate the adopted method, the energy method, for determining the damping ratio in the soil samples and to assess the effect of vertical load on the equipment damping. Energy method was used to measure the damping of soils with the Bochum resonant column device (Wichtmann et al. 2001, Wichtmann & Triantafyllidis 2013, Wichtmann et al. 2015). In the energy method, the damping ratio is determined from the hysteresis loop as the ratio of the dissipated energy in one cycle of loading ($\Delta W$) over the $4\pi$ times of the total energy ($W$). However, the device and the method of calculation of damping must be calibrated before performing a test with the device.

Aluminum sample No.4 was used to determine the equipment damping. The measured damping for aluminum sample No.4 with the energy method is presented in Figure 3.17. According to this figure, the damping in a very small strain region is more than the measured damping in a small strain region. Therefore, two additional methods, free vibration decay curves and the bandwidth method, were used to determine the damping ratio in the aluminum sample and to compare the results with the energy method.

![Figure 3.16: Measurement of damping ratio: (a) free vibration decay curve; (b) bandwidth method](image-url)
In the free vibration decay curve, after taking resonant frequency, the input power of device was disconnected and the signal received from the accelerometer (Acc.1) was recorded. The recorded signal was used to determine the damping ratio. The logarithmic decrement of, $Z$, is calculated from the recorded decay curve (Stokoe et al. 1999a):

$$Z = \ln\left(\frac{z_1}{z_2}\right) \tag{3.9}$$

where, $z_1$ and $z_2$ are the amplitude of two cycles (Figure 3.16a). Material damping ratio is calculated from the the value of $Z$ (Stokoe et al. 1999a).

$$\eta = \left[\frac{Z^2}{(4\pi^2 + Z^2)}\right]^{1/2} \tag{3.10}$$

In the second method, the bandwidth method or half-power method, the variation in RMS in respect to frequency was recorded. The recorded data were used to determine the resonant frequency and damping ratio at the desired shear strain amplitude. Resonant frequency and half-power frequencies ($f_1$ and $f_2$) are the main variables to calculate the damping ratio with this method. The frequencies $f_1$ and $f_2$ are two points, where the amplitude of excitation is equal to $1/\sqrt{2}$ of maximum amplitude (Figure 3.16b). In material with a small damping ratio, damping ratio can be estimated from the following relationship (details can be found in: Das 1993 and Stokoe et al. 1999a):

$$\eta = \frac{f_2 - f_1}{2f_r} \tag{3.11}$$

The measured damping with these methods for aluminum sample No.4 is summarized in Figure 3.17. This figure shows the impact of noise on damping at a very small strain region with the bandwidth method is less than that for the other methods which may be due to the effects of near field conditions on accelerometers. According to the observed results, two calibration lines can be defined as equipment damping in the Bochum resonant column device (Figure 3.17). The first line relates to the very small strain regions. In this region amplitude of deformations is so small (oscillation with low amplitude) that the results were affected by noise or near field conditions. The other line, which is approximately constant at the small strain region, is related to the equipment damping which is due to the connections and quality of the sample.

The equipment damping was measured under anisotropic loading by increasing the vertical load. Table 3.5 shows damping decreases with an initial increase in the vertical load at top of the sample. Connections between the aluminum sample and the device may be fitted to each other better by increasing the vertical load, therefore, the initial decrease in the damping may be due to the compliance of connections and the sample.
Figure 3.17: Equipment damping with aluminum sample, $t=4\text{cm}$

Table 3.5: The measured resonant frequency for aluminum sample under anisotropic loading

<table>
<thead>
<tr>
<th>Specimen $t$ [cm]</th>
<th>Contact size $s$ [cm]</th>
<th>Vertical load $F_v$ [N]</th>
<th>RC test $f_r$ [Hz]</th>
<th>Effects of loading bar $\Delta_v$ [%]</th>
<th>damping ratio at $\gamma \approx 10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.1</td>
<td>0</td>
<td>92.41</td>
<td>-</td>
<td>0.00205</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>500</td>
<td>92.83</td>
<td>0.45%</td>
<td>0.00184</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>1000</td>
<td>93.11</td>
<td>0.76%</td>
<td>0.00163</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>2000</td>
<td>93.19</td>
<td>0.84%</td>
<td>0.00158</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>3000</td>
<td>93.22</td>
<td>0.88%</td>
<td>0.00161</td>
</tr>
</tbody>
</table>

### 3.6 Experimental procedure

#### 3.6.1 Isotropic tests

Figure 3.18 shows the steps for assembly of the Bochum resonant column device. Firstly, specimens with 10 cm in diameter and 20 cm in height were prepared by the dry funnel deposition method for mixtures and clean coarse materials. Afterward, the top cap was...
3.6 Experimental procedure

mounted at the top of the sample. Then, the maximum vacuum of 55 kPa was applied through the top and bottom caps to stabilize the sample before assembling the resonant column device. The sample dimensions were measured accurately and then the sample was transferred to the RC device and the other parts of the device were assembled. Afterwards, the vacuum was reduced and the confining pressure was increased slowly step by step. The specimens were consolidated to the desired isotropic pressure of $\sigma_1 = \sigma_3 = 55, 80, 110, 140, 170$ and 200 kPa before performing resonant column test.

3.6.2 Anisotropic tests

Specimens with 10 cm in diameter and 20 cm in height were prepared by the dry pluviation method. The maximum vacuum of 55 kPa was applied through the top and the bottom caps to stabilize the sample. Then the vacuum was reduced and the confining pressure was increased slowly step by step. For stress induced anisotropic tests with a constant confining pressure, the confining pressure was increased up to desired value ($\sigma_3 = 200$ kPa), then the vertical pressure was increased using the double acting cylinder (Figure 3.10) slowly to: 250, 300, 350, 400, 500, 550 and 600 kPa. After consolidation of the samples at the desired stress state, resonant column tests were conducted on the sample. Then the pressure was released and the device was prepared for the new sample to perform a resonant column test for the next stress ratio. To perform the test with constant $p'$ the confining pressure was decreased and the vertical pressure was increased slowly (Table 3.6). For the stress path with constant stress ratio, confining pressure was increased to the desired isotropic pressure (55, 100, 200 and 300 kPa) and then the vertical stress was increased up to a stress ratio equal to two.

After consolidation of the sample at the desired stress conditions, the amplitude of excitation was increased to get the resonant frequency, and consequently, the stiffness and damping ratio at different amplitudes of excitation.

Table 3.6: The measured resonant frequency for aluminum sample using the resonant column device

<table>
<thead>
<tr>
<th>No.</th>
<th>stress path</th>
<th>confining pressure, $\sigma_3$, [kPa]</th>
<th>vertical pressure, $\sigma_1$, [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Isotropic</td>
<td>$\sigma_3$</td>
<td>$\sigma_3$</td>
</tr>
<tr>
<td>2</td>
<td>Constant $\sigma_3$</td>
<td>$\sigma_3$</td>
<td>$\sigma_3 + \Delta \sigma$</td>
</tr>
<tr>
<td>3</td>
<td>Constant $p'$</td>
<td>$\sigma_3 - \Delta \sigma''$</td>
<td>$\sigma_3 + 2\Delta \sigma''$</td>
</tr>
<tr>
<td>4</td>
<td>Constant $K = \sigma_3/\sigma_1$</td>
<td>$\sigma_3$</td>
<td>$\sigma_3 + \sigma_3$</td>
</tr>
</tbody>
</table>
Figure 3.18: Steps of assembly of the Bochum resonant column apparatus
3.7 Summary

The experimental program of this research was discussed in this chapter. The physical and mechanical properties of adopted materials, Hostun Sand and glass beads, were discussed in the required detail for the interpretation of the experimental results in later chapters of this thesis. The resonant column device is an accurate device but the accuracy of the results depends on the calibration of the device (Drnevich 1978). Therefore, the adopted numerical, theoretical and experimental procedures for the calibration and validation of the apparatus were discussed in this chapter. According to the analysis with ABAQUS software and experimental tests performed with aluminum samples: 1- two rotational frequencies can be detected with the free-free resonant column device, one of them is the actual rotational mode of the sample, 2- the effect of the vertical load on the dynamic behaviour of the system (resonant column device) can be ignored by minimizing the resistance moment produced at the connection area, between the loading bar and the actuator. The experimental results measured with the resonant column device were consistent with the results from the piezoelectric elements for the same sample and various stress paths. Therefore, the results showed that this device has been modified and validated successfully.
4 Effect of fines content

4.1 Introduction

Mechanical properties of geo-materials, e.g. stiffness and damping, are significantly related to the amplitude of deformations or vibrations. The small strain properties of materials can be affected by parameters which are related to the particle properties, structure of packing and boundary conditions. The structure of packing is one of the important parameters which have a significant influence on the stiffness of materials (e.g. Hardin & Drnevich 1972b, Drnevich 1978, Tatsuoka et al. 1978, Darendeli 2001, Hardin & Kalinski 2005 and Wichtmann & Triantafyllidis 2013). The structure of packing may be affected by the existence of fine particles in granular packing. The influence of fine particles on the fabric of samples is explained from a microscopic level and also empirical relationships can be modified based on the micro and macroscopic observations which is helpful to estimate the small strain stiffness of mixtures. Thevanayagam (1999) used the concept of equivalent void ratio for limited published datasets to predict maximum shear modulus and reported that the effect of $f_c$ can be captured by replacing $e$ by $e^*$ in Equation 2.7 but their data were too limited to draw conclusions. Rahman et al. (2012) estimated $e^*$ from soil grading properties and used it in Equation 2.7 to capture the effect of fines for collected data sets. They reported that $e^*$ in Equation 2.7 can capture the effect of non-plastic fines. However, collected data sets were not designed for the evaluation of $e^*$ to capture the effect of $f_c$ on $G_{max}$ and the maximum $f_c$ covered was only 20%. Thus, a suitable approach for capturing the effect of a wide range of non-plastic $f_c$ on small and intermediate strain properties is still a topic of research interest.

Soil elements have a nonlinear behaviour when they subjected to a different amplitude of deformation or vibration. The lower and upper curves for modulus ratio and damping ratio curves, proposed by Seed et al. (1984), are shown in Figure 4.1a and Figure 4.1b.
Figure 4.1: Small strain stiffness and damping in materials, modified after Darendeli (2001): (a) modulus ratio versus shear strain; (b) material damping ratio versus shear strain

From Figure 4.1a and Figure 4.1b, it can be seen that damping and modulus degradation are significantly affected by material properties, (e.g. Iwasaki & Tatsuoka 1977, Ishibashi & Zhang 1993, Darendeli 2001 and Wichtmann & Triantafyllidis 2013). Ishibashi & Zhang (1993) studied the influence of the plastic index (PI) on the nonlinear behaviour of soils using the resonant column device and concluded that the modulus ratio increases and the damping ratio decreases by increasing PI. Darendeli (2001) and Wichtmann & Triantafyllidis (2013) studied the influence of grain size distribution on the modulus degradation curve and damping. However, studies on the modulus degradation and damping ratio in granular material containing a wide range of fines content are rare. Hence, discussions on the effect of fine particles on the intermediate strain properties of granular packing containing a wide ranges of fines content are also considered in this chapter. The effect of fines on the observed experimental results will be discussed from the microscopic point of view.

Therefore, resonant column and compression wave velocity tests were conducted on the mixtures (Section 3.3) containing a wide range of fines content to evaluate the effects of fine particles on small strain properties at a macro level. Afterward, the observed results were explained through the concept of equivalent void ratio, $e^*$, developed by Thevanayagam (1998).

This chapter has four major objectives: (i) present the results from resonant column and piezoelectric element test on mixtures, covering a wide range of $f_c$, $e$ and $p'$; (ii) present
4.2 Test results on sand mixtures

4.2.1 Test results on $G_{\text{max}}$

The influence of $p'$ and $e$ on $G_{\text{max}}$ of clean Hostun Sand and sand containing fines content is presented in Figure 4.2. Test data show distinctive and separate trends of $G_{\text{max}}$ - $e$ relation for different $p'$ and $G_{\text{max}}$. These figures show $G_{\text{max}}$ increases with increasing $p'$ irrespective of $f_c$. Figure 4.2 shows the effect of void ratio on maximum shear modulus for all of the mixtures. This figure shows that the value of maximum shear modulus decreases with an increase in the void ratio. For instance, the value of $G_{\text{max}}$ for dense clean Hostun Sand (Dr=90%) is 73 MPa at the confining pressure of 55 kPa but, this value increases to 145 MPa at confining pressure of 200 kPa. Also, the value of $G_{\text{max}}$ for Hostun Sand with relative density of 90% is 145 MPa at the confining pressure of 200 kPa and decreases to 94 MPa for loose sample with relative density of 35% at the confining pressure of 200 kPa. These results confirm the dependency of $G_{\text{max}}$ on the confining pressure and density of sample which is also true for all of the mixtures (Figure 4.2). Figure 4.3 shows the effect of fines content on maximum shear modulus. This figure shows that the maximum shear modulus decreases with an increase in the fines content. As can be seen in Figure 4.3, the highest value for maximum shear modulus was obtained for clean Hostun Sand for all of the applied confining pressures. For instance, according to Figure 4.3f at $p'=200kPa$ and $e=0.72$, the value of $G_{\text{max}}$ for clean Hostun Sand is about 128 MPa. This value decreases to 84 MPa for clean Hostun Sand containing 30% $f_c$. 

a micro-mechanic analysis, Micro-CT scan, to assess the effect of $f_c$ on elastic properties; (iii) evaluate the relative performance of Hardin’s formulation for the mixtures; (iv) predict the maximum moduli of mixtures with Hardin’s empirical relation (Equation 2.7) and calculate fitting parameters for coarse particles.
Figure 4.2: The effect of confining pressure and void ratio on the maximum shear modulus of mixtures: (a) $f_c=0\%$; (b) $f_c=5\%$; (c) $f_c=10\%$; (d) $f_c=20\%$; (e) $f_c=30\%$; (f) $f_c=40\%$
4.2 Test results on sand mixtures

Figure 4.3: The effect of fines content on the maximum shear modulus for the various confining pressures: (a) $p' = 55$ kPa; (b) $p' = 80$ kPa; (c) $p' = 110$ kPa; (d) $p' = 140$ kPa; (e) $p' = 170$ kPa; (f) $p' = 200$ kPa.
4.2.2 Test results on $G(\gamma)$ and $\eta(\gamma)$

Resonant column tests were conducted to evaluate the effect of shear strain amplitude and fines content on intermediate strain properties ($G(\gamma)$ and $\eta(\gamma)$) of mixtures. Figure 4.4 shows, as an example, the effect of isotropic loading on the modulus degradation and damping curves of clean Hostun Sand. Figure 4.4a shows that shear modulus is constant and equal to the maximum shear modulus up to a shear strain amplitude of 5E-6. Also, this figure shows modulus ratio increases with an increase in the confining pressure. The effect of the confining pressure on the damping ratio is shown in Figure 4.4b. As it can be seen in this figure, the damping ratio decreases with an increase in the confining pressure.

Figure 4.5 shows the effect of fines content on modulus and damping ratio curves, in comparison with clean Hostun Sand, for samples with a relative density of 75%-77%. The general trend shows the $G/G_{max}$-log($\gamma$) shifts to the lower stiffness with an increase in the fines content up to fines content of 20% and then, $G/G_{max}$ increases for mixtures with a higher amount of fines content (Figure 4.5a). Experimental test data show that damping ratio curves have also been affected by fines content (Figure 4.5b). This figure shows the damping ratio decreases with an increase in the fines content up to fines content of 20% and it then decreases with further increases in the fines content.

Figure 4.4: The effect of $p'$ on intermediate strain properties of clean Hostun Sand Dr=38% for isotropic loading: (a) $G/G_{max}$; (b) damping ratio ($\eta$)
4.2 Test results on sand mixtures

Figure 4.5: Small and intermediate strain stiffness and damping of mixtures, Dr=80%-90%: (a) modulus degradation curves; (b) damping curves; (c) the effect of fines content on $G/G_{\text{max}}$; (d) the effect of fines content on $\eta(\gamma)$

The test data shows that the value of $\eta_{\text{min}}$ increases with an increase in the fines content. Damping and the modulus ratio at small and intermediate strain amplitude, $2E-4$ and $2E-5$, are presented in Figure 4.5d and Figure 4.5c with respect to fines content (quartz powder). These figures show the dependence of damping and the modulus ratio to fines content which is more significant in the intermediate strain region.

The test data in Figure 4.6 reveal that the shear modulus ($G$) is significantly affected by the density of the sample. This figure also shows that the shear modulus is constant and equal to the maximum shear modulus up to a shear strain less than the elastic threshold value, $\gamma_{\text{et}}$ (Stokoe et al. 1999b), which is a value between $5E-6$ for clean Hostun Sand to
1.5E-5 for clean Hostun Sand containing 40% fines content. The value of $\gamma_{et}$ increases significantly with an increase in the fines content. The general relation between $G/G_{max}$ and $\log \gamma$ curves for the mixtures at the mean effective stress of 200 kPa is presented in Figure 4.7. This figure shows that the effect of density of sample on $G/G_{max}$ is not significant for all of the mixtures. Figure 4.8 shows the effect of void ratio on damping ratio curves. The presented experimental results show the insignificant effect of relative density on the damping ratio and $G/G_{max}$ of clean Hostun Sand and mixtures which is in line with the experimental results presented by Kokusho (1980), Iwasaki et al. (1978) and Wichtmann & Triantafyllidis (2013). This figure reveals that $\gamma_{et}$ decreases slightly with an increase in the fines content up to $20\% f_c$, and it then increases significantly with an increase in the $f_c$. The test data show that the damping ratio increases with an increase in the fines content up to $f_c$ of 20% and then decreases with further increases in the fines content.
Figure 4.6: Shear modulus versus shear strain, $p'=200$ kPa: (a) $f_c=0$; (b) $f_c=5$%; (c) $f_c=10$%; (d) $f_c=20$%; (e) $f_c=30$%; (f) $f_c=40$%
Figure 4.7: Modulus ratio \( \frac{G}{G_{\text{max}}} \) versus shear strain, \( p'=200 \text{ kPa} \): (a) \( f_c=0 \); (b) \( f_c=5\% \); (c) \( f_c=10\% \); (d) \( f_c=20\% \); (e) \( f_c=30\% \); (f) \( f_c=40\% \)
Figure 4.8: Damping ratio versus shear strain, $p' = 200$ kPa: (a) $f_c = 0$; (b) $f_c = 5\%$; (c) $f_c = 10\%$; (d) $f_c = 20\%$; (e) $f_c = 30\%$; (f) $f_c = 40\%$
4.3 Test results for glass bead mixtures

Resonant column tests and compression element tests were conducted on coarse glass beads and mixtures to assess the effect of fines content on stiffness (Goudarzy et al. 2014). Resonant column and compression element tests were carried out at isotropic pressures of 50, 100, 150 and 200 kPa. After increasing the isotropic pressure to the target isotropic pressure, a rest period of at least 1 hour was applied to allow for creep of the sample. The structure of adopted samples was susceptible to significant volume change and collapse during the loading which was due to the interaction between the highly rounded fine and coarse particles (Yang & Wei 2012). The results for samples without collapse are presented in this section. The maximum shear modulus was determined by the resonant frequency measured using the resonant column device. In the compression wave velocity test a sinusoidal wave with a frequency of 10 kHz was transmitted by the compression elements which were mounted in the top and bottom caps of the sample. The first arrival or deflection method was adopted to detect travel time, \( t_p \) (e.g. Yamashita et al. 2005). Therefore, the compression wave velocity \( v_p \) is equal to the length of the sample at the time of the compression element test over the travel time. Therefore, the values of \( G_{\text{max}} \) and \( M_{\text{max}} \) were measured using the experimental results and the value of Young’s modulus, \( E_{\text{max}} \), was calculated using Equation 4.1 (Birch 1961 and Mavok et al. 2009).

\[
E = \frac{G(3M - 4G)}{M - G}
\]

where \( M \) is the p-wave modulus, \( G \) is the shear modulus and \( E \) is Young’s modulus. The experimental results show the increasing and decreasing of the maximum elastic moduli, \( G_{\text{max}}, M_{\text{max}} \) and \( E_{\text{max}} \), with increase in the mean effective stress and void ratio respectively (Figure 4.9). As an example, Figure 4.9a shows the effect of \( p' \) on the maximum shear modulus, \( G_{\text{max}} \). As can be seen in this Figure, \( G_{\text{max}} \) increases with an increase in \( p' \) and decreases significantly with an increase in \( e \). Also, the results demonstrate the dependence of maximum elastic moduli to fine particles (e.g. Figure 4.9b, Figure 4.9c and Figure 4.9d). Test data show that the value of the maximum elastic moduli decreases with an increase in the fines content up to 30% fines content and then it increase with further increasing of fines content which will be explained from the microscopic point of view in the following sections.
4.4 Interpretation based on the micro-structure

The conceptual model developed by Thevanayagam (1998) is used to explain the influence of fine particles on the mechanical response of coarse particles at microscopic and macroscopic scales. Figure 4.10 shows the effect of fine particles on force chains and consequently the stiffness of mixtures. Based on the amount of fines content and the situation of fine particles in the mixture, five cases can be distinguished. In case i: in this case, fines content is zero and coarse particles make up the packing structure.

Figure 4.9: (a) the effect of void ratio and \( p' \) on \( G_{\text{max}} \) of coarse glass beads; (b) \( G_{\text{max}} \) vs \( e \) for the mixtures at \( p'=200 \) kPa; (c) \( M_{\text{max}} \) vs \( e \) for the mixtures at \( p'=200 \) kPa; (d) \( E_{\text{max}} \) vs \( e \) for the mixtures at \( p'=200 \) kPa;
By increasing the fines content, fines are placed in the coarse matrix. Due to the particle size ratio, fine particles can be accommodated in the void spaces and fines are confined by coarse particles (case ii), therefore they do not increase the amount of inter-particle
contacts and so do not contribute in the force chains of granular structure. However, some times, due to the sample preparation methods or material properties, fine particles are accommodated between two coarse particles (case iii and case iv). In this situation, fines separate two coarse particles from each other and due to the interaction between fine and coarse particles; stiffness of packing will be significantly affected. For ease of discussion, Figure 4.11 is used to explain the effect of particle size or interaction between particles on shear stiffness. From literature (e.g. Duffy & Mindlin 1957, Petrakis & Dobry 1987, Walton 1987, Chang & Liao 1994, Bui 2009 and Otsubo et al. 2015), grain stiffness and contact stiffness are two main characteristics which can make a significant effect on shear wave velocity and shear stiffness in granular packings. In the packing containing the same particle characteristics, shear wave velocity and shear stiffness are strongly affected by contact properties or contact stiffness ($K_{contact}$, Figure 4.11). Contact stiffness ($K_{contact}$) between smooth spheres is determined using Hertzian theory (Section 2.4). Hertzian theory reveals that normal contact force, contact radius and contact friction increases proportionally to grain diameter (Johnson 1985). Therefore, the observed decreasing in stiffness for mixtures containing fines (case iii and case iv) could be explained through the weak contact between fine and coarse particles in comparison with a contact between coarse and coarse particles.

By further increasing the fines content, fine particles will be dominant in the response of granular packing (case v); in this case, coarse particles will act like reinforced elements inside fine particles packing. Therefore, displacement and sliding of fines controlled by coarse particles, and the stiffness of packing depends on the characteristics of the fine particles. Thevanayagam (1998) proposed parameters $b$ and $m$ to determine the influences of fine particles at microscopic level (Section 4.5). As long as fines content are less than a certain value (threshold fine content, $f_{thr}$), parameter $b$ will measure the active contacts (case iii and case iv) and for fines content greater than the threshold value parameter $m$ will evaluate the interaction between coarse and fine particles (case iv).
To evaluate the effect of $f_c$ on the micro-structure of Hostun Sand with $f_c$, Micro-CT scans on loose and dense specimens were carried out at Kumamoto University. The size of specimens was 40mm in height and 7mm in diameter and was prepared by the same dry deposition method. Figure 4.12a and Figure 4.12b show the Micro-CT images of the cross sections of specimens for sand with 10% and 50% $f_c$ for relative densities of 92% and 85% respectively. The white portions are sand particles and the grey zones are fines, i.e. quartz powders. It is apparent from Figure 4.12a that 10% $f_c$ merely coated some of the coarse particles. A fraction of $f_c$ were located between coarse particles and the remaining fraction of fines were placed in void spaces between the sand particles. However, some coarse particles were still in direct contact. This confirms that the micro-structure for sand with 10% fines is still dominated by sand structure, although some fines are active in contact. The Micro-CT scan for clean Hostun Sand mixed with 50% $f_c$ is presented in Figure 4.12b. It is evident that the shapes of sand particles are lost and they are floating on gray zones, i.e. in $f_c$. Thus, the stiffness of the specimen would be dominated by $f_c$. This micro-structure was also explained as “sand-in-fines”.

Micro-CT scanning was also done on the glass bead mixtures. The imaging method (Micro-CT scan) was done on samples with 1 cm in diameter and 2 cm in height prepared by tamping methods, the same as the method used for preparing the glass bead mixtures in the resonant column tests. It is worthwhile to mention that the goal of such scanning was to find the effect of sample preparation on the position of fines particle. Figure 4.13 is an example of Micro CT scans of adopted mixtures (30 and 50 percentage of fines content).
AVISO software was used to count the number of coarse particles and contacts between two coarse particles. For decreasing the time of analysis, two elements, containing 40 coarse particles, were chosen from the samples with 0 and 20 percentage fines content (Figure 4.14a and Figure 4.14b). Analyses of samples show that the number of contacts between coarse particles decreased with an increase in the fines content. For example, the number of contacts in the specimen containing 40 coarse particles and 0% fines content was 77 which decreased to 42 for the specimen containing 43 coarse particles and 20% fines content (specimens with the same number of coarse particles). For ease of discussion, the term of coarse-coarse coordination number, \( CN_{\text{coarse-coarse}} = \frac{2N_{b,\text{coarse}}}{N_{b,\text{coarse}}} \).
(Shire et al. 2014) is used for comparing the results. In this relationship, $N_{c, \text{coarse-coarse}}$ is the number of contacts between coarse particles and $N_{b, \text{coarse}}$ is the number of coarse particles. Therefore, in the sample containing 0% $f_c$, the value $CN_{\text{coarse-coarse}}$ was equal to 3.83 and for sample containing 20% $f_c$, $CN_{\text{coarse-coarse}}$ was 1.95 which shows the decreasing of $CN_{\text{coarse-coarse}}$ with an increase in fines content. This means, due to the sample preparation some of the fines were placed between the coarse particles. Therefore, for fines content less than threshold value ($f_c < f_{\text{thr}}$), micro CT images show that some of the fine particles have been located between coarse particles. Therefore, fines particles participate in force chains and their contribution must be taken into account. Therefore, stiffness of packing is affected by fines content less than threshold value, the same as the observed experimental results. However, for fines content greater than threshold value ($f_c > f_{\text{thr}}$), fine particles will be dominant on coarse particles, therefore, coarse particles act like a reinforced element inside fine materials (Figure 4.13d and Figure 4.13c). This confirms the model of “fines-in-sand” by Thevanayagam et al. (2002) that some fine particles are active in sand force structures and others are inactive. Thus, for the same void/solid ratio, i.e. same $e$, the effective particle contact decreases with an increase in $f_c$ and, therefore, their stiffness would be less than for clean sand for the same $e$. This is also consistent with experimental observation as noted above. However, this also means that for the same $e$, sand with fines does not represent the same force structure as clean sand, and thus the concept of “equivalent” void ratio for sand with $f_c$ emerged.

Figure 4.14: Analysis of Micro-CT images: (a) clean coarse glass beads; (b) clean coarse glass beads containing 20% fine particles
4.5 Concept of equivalent granular void ratio, $e^*$

The concept of “equivalent” void ratio was first presented by Mitchell (1976) who considered fines as void in force structure. For the “fines-in-sand” micro-structural model, Thevanayagam et al. (2002) recognized the active contribution of a fraction of fines particles in sand force structure and proposed Equation 4.2.

$$e^* = \frac{e + (1-b)f_c}{1-(1-b)f_c}$$  \hspace{1cm} (4.2)

where, $e^*$ is the equivalent granular void ratio and $b$ is the fraction of fines that are active in the sand force structure. In recent years, interest has been growing in the use of the equivalent void ratio to characterize the behaviour of sand-fines mixtures. The key step in doing that is the determination of factor $b$ in Equation 4.2. Most studies have employed the best fit approach to obtain the $b$ value. The value of $b$ varies from 0 to 1. $b=0$ means fines are inactive (i.e. void) in the sand force structure which is a very common assumption for very low $f_c$. However, for higher $f_c$, $b \neq 0$ which means $b$ is a function of $f_c$. From the mathematical attributes of binary packings, Rahman et al. (2008) developed a semi-empirical relation to predict the parameter $b$.

$$b = \left[1 - \exp\left(-\lambda \frac{f_c/f_{thr}}{k}\right)^{n_b}\right] \left(\frac{f_c/f_{thr}}{r}\right)^r$$ \hspace{1cm} (4.3)

where $r = (D_{10}/d_{50})^{-1}$, $D_{10}$ = size of sand at 10% finer, $d_{50}$ = size of fine at 50% finer, $k = (1-r^{0.25})$, $\lambda$ and $n_b$ are the fitting parameters which depend on the soil properties. $f_{thr}$ = threshold fine content which separates “fines-in-sand” from the “sand-in-fines” microstructure. In this study, $f_{thr}$ was estimated from the $e_{min}$ versus $f_c$ curve. Furthermore, the experimental results from damping ratio and modulus ratio curves revealed that the behaviour of mixtures changed with fines more than 30%. Regarding these evidence, $f_{thr}$ was between 20\% to 30\% which was assumed to be 25\% in this study. It is noted that $b$ can be obtained from back analysis (Thevanayagam et al. 2002; Ni et al. 2005), however to take advantage of the prediction of $b$, the above approach is used in this study.

For the “sand-in-fines” micro-structure when $f_c \geq f_{thr}$, Thevanayagam (1998) proposed Equation 4.4 to determine $e^*$.

$$e^* = \frac{e}{f_c + \frac{1-f_c}{R_d}}$$ \hspace{1cm} (4.4)

Where, $m$ is a fitting parameter that evaluates the influence of coarse particles inside the fine particle matrix and $R_d$ is size ratio, $\frac{D_{50}}{d_{50}}$ (Ni et al. 2005).

It is worthwhile to mention that the current study does not intend to establish $e^*$ concept.
which was done earlier (Thevanayagam 1998; Rahman et al. 2008), but offers a simple and unified methodology to predict $G_{\text{max}}$ irrespective of $f_c$ which may trigger significant research interest along this line for this challenging problem.
4.6 Empirical relationships for Hostun Sand mixtures

4.6.1 Empirical relationships to predict $G_{max}$

Various prediction models have been developed in the last few decades which are applicable to computer programs for geo-material response analysis. Hardin & Black (1966) were, arguably, the first to propose one of the most widely used empirical relationship, which considers the effect of density through the void ratio, $e$, and $p'$ to predict $G_{max}$ of a soil. This relationship is referred to as Hardin’s relation which is represented by the following general form (Equation 4.5):

$$G_{max} = kf(e) \rightarrow G_{max} = Af(p)f(e) \rightarrow G_{max} = Ap_a\left(\frac{p'}{p_a}\right)^n f(e)$$  (4.5)

where, $A$ is a material constant which depends on the soil type, $p_a$ is the atmospheric pressure (100 kPa), $p'$ is the mean effective stress, $n$ is an exponent and $f(e)$ is the void ratio function. Two common functional forms for $f(e)$, Equation 4.6 (Hardin & Black 1966) and Equation 4.7 (Jamiolkowski et al. 1995), are usually found in the literature (Table 2.1).

$$f(e) = \frac{(x - e)^2}{1 + e}$$  (4.6)

$$f(e) = e^{-y}$$  (4.7)

where, $x$ depends on the angularity of soils (e.g. $x=2.97$ for angular sands and 2.17 for rounded sands). Note, $x$ is a limiting void ratio in Equation 4.6; for example, if $e$ is assumed as a value equal to 2.17, for rounded sands $f(e)$ will be equal to zero and consequently, $G_{max}$ will be 0. Therefore, to avoid this limitation, $x$ can be considered as a fitting parameter which may depend on the angularity of soils. $y$ in Equation 4.7 is also a fitting constant. Apart from Hardin and Jamiolkowski’s functional forms of $f(e)$, Seed’s model has $K_{2,\text{max}}$ parameter (Seed et al. 1984). Seed et al. (1984) proposed a relationship between $G_{max}$ and $p'$ as $G_{max} = 218.8 \ K_{2,\text{max}} (p')^{0.5}$ (in SI unit); where, $K_{2,\text{max}}$ may be a function of $e$. $K_{2,\text{max}}$ versus $e$ for different $f_c$ is presented in Figure 4.15; where $K_{2,\text{max}}$ decreased with $f_c$. The results show that the value of $K_{2,\text{max}}$ not only depends on the density of materials but also depends on the fines content. Therefore, $K_{2,\text{max}}$, in Seed’s relationship, must be correlated with the fines content.
Figure 4.15: The effect of fines content on $K_{2\text{max}}$ for the various confining pressures: (a) $p'=55$ kPa; (b) $p'=80$ kPa; (c) $p'=110$ kPa; (d) $p'=140$ kPa; (e) $p'=170$ kPa; (f) $p'=200$ kPa
4.6.2 Inspection of Hardin’s relationship

To predict the maximum shear modulus of mixtures, the fitting parameters of Hardin’s relation must be determined for all of the mixtures. As a first step, the constant fitting parameters of void ratio functions, \( x \) and \( y \), must be determined accurately. The values of \( x \) and \( y \) were determined by fitting the curve \( k \cdot f(e) \) to the test data in \( G_{\text{max}} \) versus \( e \) curves. Figure 4.16 shows an example of the fitted curves to \( G_{\text{max}} \) versus \( e \) for clean Hostun Sand. The trend lines are plotted in this figure as a form of \( k \cdot f(e) \), where \( f(e) \) is the void ratio function (e.g. Equation 4.6 and Equation 4.7) and \( k \) is the remaining part of Equation 4.5 which depends on \( p' \) and the adopted \( f(e) \). By preforming this calibration analysis, the constant values of \( x \) and \( y \) were determined. The solid lines in Figure 4.16 are related to Hardin’s void ratio function, Equation 4.6, and the dashed lines are related to Jamiolkowski’s void ratio function, Equation 4.7. Afterwards, \( G_{\text{max}} \) was normalized with respect to the void ratio function. Figure 4.17a and Figure 4.17b show an example of the normalized \( G_{\text{max}}/p' / p_a \) for clean Hostun Sand for both of the void ratio functions. These curves are useful to find the value of the parameter \( A \) and exponent of pressure function, \( n \), in Hardin’s relation (Equation 4.5). The values of the fitting parameters in Hardin’s relation for clean Hostun Sand are summarized in Table 4.1.

![Figure 4.16: Gmax versus e to determine the constant fitting parameters of the void ratio function for clean Hostun Sand](image-url)
Figure 4.17: The fitting parameters of Hardin’s relation to determine $G_{max}$ of clean Hostun Sand: (a) Hardin’s void ratio function; (b) Jamiolkowski’s void ratio function

Table 4.1: Summary of fitting parameters for Hardin’s relation

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<th>Dr [%]</th>
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<th>$A$</th>
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<th>$A$</th>
<th>$R^2$</th>
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</tbody>
</table>

Two statistical measures of Root-Mean-Square-Deviation (RMSD) and $R^2$ were used to estimate the scatter of the results in Table 4.1. The RMSD and $R^2$ for Hardin’s $f(e)$ were 2.83 and 0.981 and for Jamiolkowski’s $f(e)$ were 2.97 and 0.978 respectively. Therefore, the results for both $f(e)$ showed a good agreement between the observed and predicted results.

The same analyses were performed on the other mixtures to determine the fitting parameters of Hardin’s relation. Figure 4.18a shows an example of fitted $k_f(e)$ curves to the test data for mixture containing 30% fines content to find the constant value of $x$ in the void ratio function (Equation 4.6). Figures 4.18b, 4.18c and 4.18d show the effect of fines content on the constant fitting parameters of clean Hostun Sand mixed with quartz powder. As can be seen in Figure 4.18, the fitting parameters of Hardin’s relation and the void
ratio function (Equation 4.5 and Equation 4.6) are affected by fines content significantly. Therefore, for the prediction of the maximum shear modulus, using Hardin’s relation, the fitting parameters must be determined for all of the mixtures which is beyond the scope of our study. In this chapter, instead of calculating the fitting parameters of Equation 4.5, Equation 4.6 and Equation 4.7 for mixtures, the concept of the equivalent void ratio, $e^*$, will be used to estimate the value of $G_{max}$ for all of the mixtures.

\[
f_c = 30\% \\
k_f(e) = k(x - e)^2(1 + e) \\
x = 4.06
\]

\[
p' = 55 \text{ kPa} \\
p' = 80 \text{ kPa} \\
p' = 110 \text{ kPa} \\
p' = 140 \text{ kPa} \\
p' = 170 \text{ kPa} \\
p' = 200 \text{ kPa}
\]

**Figure 4.18:** (a) $G_{max}$ versus $e$ to determine the value of $x$ in Equation 4.7; (b) the effect of $f_c$ on the value of $x$ in Equation 4.7; (c) the effect of $f_c$ on the stress exponent, $n$, in Equation 4.5; (d) the effect of $f_c$ on the constant value of $A$ in Equation 4.5
4.6.3 $e^*$ in Hardin’s relationship

4.6.3.1 Hardin’s relationship for mixtures

Micro-structural studies suggest that the transition soil’s behaviour is controlled by sand structure for $f_c < f_{thr}$ and fine structure for $f_c > f_{thr}$, so Hardin’s relationship for clean Hostun Sand should capture the effect of $f_c$ through $e^*$. Thus, Hardin’s relation was established first for clean Hostun sand for both functional forms of $f(e)$. The fitting parameters, $A$, $n$ and $x$ or $y$ for Hardin’s relation, are presented in Table 4.1.

According to the micro-structural analysis, the effect of $f_c$ on $G_{max}$ can be captured with $e^*$ while the other parameters remain the same as determined values for clean Hostun Sand. This means that substitution of $e^*$ for $e$ in Equation 4.5 for clean sand should capture the effect of $f_c$. Thus, the equation Equation 4.5 becomes:

$$G = kf(e^*) \rightarrow G = Af(p)f(e^*) \rightarrow G_{max} = Ap_a \left( \frac{P}{P_a} \right)^n f(e^*)$$

(4.8)

where, $A$ and $n$ are parameters for clean Hostun Sand. Other parameters, $x$ and $y$ in Equation 4.6 and Equation 4.7, also remain the same as determined values for clean Hostun Sand. The accuracy of the predicted results depends on the micro-structural parameter $b$, i.e. $\lambda$, $n_b$ and $m$ in the $e^*$ relationship.

4.6.3.2 Calibration of parameters

Giving consideration to a large number of data sets, Rahman et al. (2008) proposed $\lambda$ and $n_b$ are equal to 0.3 and 1, respectively, which capture many characteristic responses of undrained soil behaviours through $e^*$ (e.g. Rahman et al. 2008, Rahman & Lo 2012 and Rahman & Lo 2014). However, Rahman et al. (2008) suggested that these parameters may vary for different type of soils. The other micro-structural parameters for Hostun Sand containing fines are available from grading properties. For $f_c > f_{thr}$, $m$ is needed for Equation 4.4. $m$ was determined by back analysis of the test data which were 0.11 and 0.14 for Hostun Sand containing 30 and 40 percent fine content respectively. Ni et al. (2005) showed that $m$ can be a function of $\frac{C_{uc}C_{uf}^2}{R_d}$ where, $C_{uc}$ and $C_{uf}$ are the uniformity coefficient of coarse and fines respectively. The relationship between $m$ and $\frac{C_{uc}C_{uf}^2}{R_d}$ was also approximated by a linear relationship (Figure 4.19) for the mixtures used in this study. According to the grain size distribution of Hostun Sand and quartz powder, the value of $\frac{C_{uc}C_{uf}^2}{R_d}$ for the adopted materials was about 0.32. Thus, $m = 0.12$ was obtained from the linear relation for Hostun Sand for $f_c > f_{thr}$.
Five different calibrations were performed to evaluate the influence of parameters $b$ and $m$ on the accuracy of the predicted $G_{\text{max}}$ for clean Hostun Sand containing different percentages of fines content. Parameter $b$ is related to the fitting parameters of $\lambda$, therefore, calibrations were performed on the constant fitting parameter of $\lambda$. Table 4.2 presents a summary of the calibrations performed in this chapter to find the accuracy of the predicted results using Equations 4.2 and 4.4.

- **Calibration I**

In this calibration, “fines-in-sand” micro-structure is assumed for all $f_c$ and the fitting parameters, as proposed by Rahman et al. (2008) (i.e. $\lambda=0.30$ and $n_b=1.0$), were used to predict $G_{\text{max}}$ for $f_c=0$-40%, despite $f_{\text{thr}}=25\%$. Figure 4.20a and Figure 4.20b show $G_{\text{max}}$ vs $e^*$ for the observed results at $p'$ of 55kPa and 200kPa (only the lower and upper range of $p'$ were plotted for clarity).

The results, in comparison to Figure 4.3f, show that the influence of $f_c$ can be captured by the concept of $e^*$. The dashed lines represent the fitted curves for Hardin’s $f(e)$ and the solid lines are the fitted curves for Jamiolkowski’s $f(e)$. All other fitting parameters are the same as clean Hostun Sand (Table 4.1). However, the measured $G_{\text{max}}$ for $f_c<30\%$ are close to both the fitting lines, but deviation was observed for $f_c>30\%$. 

---

**Figure 4.19:** Parameter $m$ versus $\frac{C_{\text{uc}}C_{\text{uf}}^2}{R_d}$, modified after Ni et al. (2005)
Table 4.2: Summary of calibrations performed on parameters \( \lambda \) and \( m \) for Jamiolkowski’s \( f(e) \)

<table>
<thead>
<tr>
<th>Calibration</th>
<th>( f_c &lt; f_{thr} )</th>
<th>( f_c &gt; f_{thr} )</th>
<th>( R^2 )</th>
<th>RMSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration I</td>
<td>Eq. 4.3, ( \lambda=0.3 )</td>
<td>Eq. 4.3, ( \lambda=0.3 )</td>
<td>0.86</td>
<td>8.92</td>
</tr>
<tr>
<td>Calibration II</td>
<td>Eq. 4.3, ( \lambda=0.3 )</td>
<td>Eq. 4.4, ( m ) from Fig. 4.19</td>
<td>0.97</td>
<td>3.58</td>
</tr>
<tr>
<td>Calibration III</td>
<td>Eq. 4.3, ( \lambda=\text{back analysis} )</td>
<td>Eq. 4.4, ( m ) from Fig. 4.19</td>
<td>0.98</td>
<td>3.15</td>
</tr>
<tr>
<td>Calibration IV</td>
<td>Eq. 4.3, ( \lambda=\text{back analysis} )</td>
<td>Eq. 4.4, ( \text{m}=\text{back analysis} )</td>
<td>0.98</td>
<td>3.23</td>
</tr>
<tr>
<td>Calibration V</td>
<td>Eq. 4.3, ( \lambda=\text{back analysis} )</td>
<td>Eq. 4.4, ( \text{m}=\text{back analysis} )</td>
<td>0.98</td>
<td>2.71</td>
</tr>
</tbody>
</table>

Figure 4.20: \( G_{\text{max}} \) versus \( e^* \) for calibration I: (a) \( p'=55 \) kPa; (b) \( p'=200 \) kPa

Figure 4.20 also shows all data points and the scatter of the predicted and measured \( G_{\text{max}} \). This figure shows that the scatter of data is not acceptable for \( f_c>30\% \). The overall \( R^2 \) and RMSD for all data points were 0.86 and 8.92 for Hardin’s \( f(e) \). However, overall \( R^2 \) and RMSD were 0.95 and 4.65 for test data with \( f_c<30\% \). These results confirm that the “fines-in-sand” model, i.e. Equation 4.2, is suitable for \( f_c<f_{thr} \) and it is not applicable for \( f_c>f_{thr} \).

- **Calibration II**

In this calibration, Equation 4.2 and Equation 4.3, with the fitting parameters proposed by Rahman et al. 2008 (i.e. \( \lambda=0.30 \) and \( n_b=1.0 \)), were used to predict \( G_{\text{max}} \) of clean Hostun Sand containing up to 20\% \( f_c \). Thus, only grading parameters, i.e. \( D_{10} \) and \( d_{50} \), were required to calculate \( e^* \).
Equation 4.4 was employed to predict $G_{\text{max}}$ of clean Hostun Sand with $f_c \geq 30\%$. The grading properties, i.e. $D_{50}$, $d_{50}$ and $m=0.12$ from the linear relationship in Figure 4.19, were required to calculate $e^*$. Therefore, this calibration was not dependent on back analysis which requires test data as a prerequisite. Figure 4.21a and Figure 4.21b show $G_{\text{max}}$ versus $e^*$, at the confining pressure of 55 and 200kPa. The deviation of the measured $G_{\text{max}}$ and fitted curves for both $f(e)$ reduced significantly, particularly after $f_c \geq 30\%$. The functional form of Hardin’s $f(e)$ and Jamiolkowski’s $f(e)$ for clean Hostun Sand are...
presented with dotted and solid lines respectively. The data at higher \(e^*\) and lower \(G_{\text{max}}\) show a relatively higher scatter than at lower \(e^*\) and higher \(G_{\text{max}}\).

Hardin’s relation also shows a slight deviation toward a higher \(e^*\) and lower \(G_{\text{max}}\) which may be attributed to the inherent variability of \(G_{\text{max}}\) toward that end. Therefore, to have an objective comparison, \(G_{\text{max}}\) was predicted for \(f_c\) with Hardin’s relation with the same constant parameters as determined for clean Hostun Sand and \(e\) replaced by \(e^*\).

The predicted versus measured \(G_{\text{max}}\) for Hardin’s \(f(e)\) and Jamiolkowski’s \(f(e)\), for all of \(f_c\), are shown in Figure 4.22a and Figure 4.22b respectively. Figure 4.22a and Figure 4.22b show a good agreement between the predicted and measured \(G_{\text{max}}\) for all data points. The values of \(R^2\) and RMSD of predicted results are summarized in Appendix A (Table A.1 and Table A.2) for Hardin’s and Jamiolkowski’s \(f(e)\) respectively. The overall \(R^2\) and RMSD were 0.94 and 5.18 for Hardin’s \(f(e)\) and 0.97 and 3.58 for Jamiolkowski’s \(f(e)\) respectively. It is worthwhile to mention that the fitting parameters of Hardin’s and Jamiolkowski’s void ratios are the same as determined values for clean Hostun sand. It is noted that Jamiolkowski’s \(f(e)\) with an inverse power function of \(e\) gave better prediction than Hardin’s \(f(e)\) for a large data set with a wide range of \(f_c\) and the RMSD of 3.58 is used as a reference value to evaluate the performance of other calibrations.

The fitting parameters for this calibration were obtained from soil grading properties. Also, constant parameters in Hardin’s relation (Equation 4.5) are the same as the determined values for clean Hostun Sand (Table 4.1). Therefore, this calibration was not dependent on the back analysis of the test data. Thus, the RSMD of this calibration for Jamiolkowski’s \(f(e)\), 3.58, was used as a reference value to evaluate the accuracy of other calibrations. The reduction of RMSD for Jamiolkowski’s \(f(e)\) from calibration I to calibration II is about 23% of 4.65.

- **Calibration III**

In this calibration, Equation 4.3 was used to determine the value of parameter \(b\) for clean Hostun Sand containing 0 to 20% \(f_c\). However, the value of parameter \(\lambda\) in Equation 4.3 was determined by back analysis of test data to get the maximum \(R^2\). \(m=0.12\), as in Figure 4.19, was used to find \(e^*\) with Equation 4.4 for clean Hostun Sand with 30-40% fines content. \(G_{\text{max}}\) vs \(e^*\) is presented for \(p'=55\) and 200 kPa in Figure 4.23a and Figure 4.23b. The predicted vs measured \(G_{\text{max}}\) for all data points are shown in Figure 4.24a and Figure 4.24b, and their \(R^2\) and RMSD are summarized in Appendix A (Table A.1 and Table A.2) for Hardin’s and Jamiolkowski’s \(f(e)\) respectively. The overall \(R^2\) and RMSD for all data points were 0.96 and 4.29 for Hardin’s \(f(e)\) and 0.98 and 3.15 for Jamiolkowski’s \(f(e)\) respectively.
4.6 Empirical relationships for Hostun Sand mixtures

Figure 4.23: $G_{\text{max}}$ versus $e^*$ for calibration III: (a) $p'=55$ kPa; (b) $p'=200$ kPa

Figure 4.24: Measured $G_{\text{max}}$ versus predicted $G_{\text{max}}$ for calibration III: (a) Hardin’s $f(e)$; (b) Jamiolkowski’s $f(e)$

Again, Jamiolkowski’s $f(e)$ gives better prediction than Hardin’s $f(e)$. The reduction of RMSD for Jamiolkowski’s $f(e)$ is only about 12% of 3.58 obtained from calibration II, although a large fraction of data points used in the back analysis to get the value of $\lambda$ in Equation 4.3.
Calibration IV

In this calibration for $f_c < f_{thr}$, $\lambda=0.30$ as proposed by Rahman et al. 2008 was used in Equation 4.3. However, for $f_c > f_{thr}$, $m$ was determined by back analysis for the maximum $R^2$. $m$ for 30% and 40% $f_c$, denoted as $m_{30}$ and $m_{40}$, were variable for different $p'$.

Figure 4.25: $G_{max}$ versus $e^*$ for calibration IV: (a) $p'=55$ kPa; (b) $p'=200$ kPa

Figure 4.26: Measured $G_{max}$ versus predicted $G_{max}$ for calibration IV: (a) Hardin’s $f(e)$; (b) Jamiolkowski’s $f(e)$
Figure 4.25a and Figure 4.25b show $G_{\text{max}}$ vs $e^*$ only for $p' = 55$ and 200 kPa. Although it may not be visually recognizable, the data points for $f_c > f_{\text{thr}}$ are closer to fitting curves in comparison to calibration II.

The plot for predicted vs measured $G_{\text{max}}$ for all data points are shown in Figure 4.26a and Figure 4.26b. Again, Jamilokowski’s $f(e)$ gave a better prediction than Hardin’s $f(e)$. The reduction of RMSD for Jamilokowski’s $f(e)$ is only about 10% of 3.58, although a fraction of data used in the back analysis to obtain $m$.

- Calibration V

In calibration V, $\lambda$ in Equation 4.3 was determined by back analysis of the test data where $f_c < f_{\text{thr}}$. For $f_c > f_{\text{thr}}$, $m$ value was determined by back analysis of test data ($m_{30}$ and $m_{40}$). Figure 4.27 shows $G_{\text{max}}$ vs $e^*$ for all of the applied confining pressures. As can be seen in Figure 4.27, the effect of $f_c$ can be captured by $e^*$ for all of the confining pressures. The scatter of the data points are lowest compared to all other calibrations. The same as other calibrations, Jamilokowski’s $f(e)$ gives better prediction than Hardin’s $f(e)$. A good agreement between the predicted and measured $G_{\text{max}}$ was confirmed as shown in Figure 4.28a and Figure 4.28b. The values of $R^2$ and RMSD are presented in Appendix A (Table A.3 and Table A.4). The reduction of RMSD for Jamilokowski’s $f(e)$ is about 24% of 3.58 confirming the best prediction among all calibrations. Figure 4.32 shows the effect of $e^*$ and $p'$ on the maximum shear modulus for all of the mixtures.

Figure 4.27: $G_{\text{max}}$ versus $e^*$ for calibration V at (a) $p'=55$ kPa and (b) $p'=200$ kPa
4.7 Empirical relationships for glass bead mixtures

4.7.1 Hardin’s relation

In this section the fitting parameter of Hardin’s relation will be determined for coarse glass beads. Jamiolkowski’s void ratio function (Equation 4.7) has been used to discuss the results of glass bead mixtures at a macro level. As a first step, the fitting parameter of Jamiolkowski’s void ratio function, $y$, must be determined accurately. The value of $y$ was determined by fitting the curve $k f(e)$ to the test data in Figure 4.29a. The trend lines are plotted in this figure as a form of $k f(e)$; where $f(e)$ is the void ratio function and $k$ is the remaining part of Equation 4.5 which depends on the value of $p'$. The constant value of $y$ was determined after performing a regression analysis. The dashed lines in Figure 4.29a are related to the Jamiolkowski’s void ratio function. Afterwards, $G_{max}$ was normalized with respect to the void ratio function (Figure 4.29b). The normalized $G_{max}$ was drawn versus $p'/p_a$. This curve was useful to find the value of parameter $A$ and the exponent of stress ($n$) in Hardin’s relation (Equation 4.5) for coarse glass beads. The same analysis were conducted to find the constant fitting parameters of Hardin’s relation to estimate the values of $M_{max}$ and $E_{max}$. Table 4.3 presents the values of the fitting parameters in Hardin’s relation for coarse glass beads from the performed analyses.
4.7 Empirical relationships for glass bead mixtures

![Graphs showing empirical relationships for glass bead mixtures]

Figure 4.29: Determination of fitting parameters of Hardin’s relation for coarse glass beads: (a) $G_{\text{max}}$ vs $\varepsilon$ and (b) $G_{\text{max}}/f(\varepsilon)$ vs $p'/p_a$

<table>
<thead>
<tr>
<th>Modulus</th>
<th>Dr [%]</th>
<th>$y$</th>
<th>A</th>
<th>n</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{\text{max}}$</td>
<td>40-80</td>
<td>3.98</td>
<td>0.180</td>
<td>0.403</td>
<td>0.98</td>
</tr>
<tr>
<td>$M_{\text{max}}$</td>
<td>40-80</td>
<td>1.869</td>
<td>2.002</td>
<td>0.301</td>
<td>0.97</td>
</tr>
<tr>
<td>$E_{\text{max}}$</td>
<td>40-80</td>
<td>3.99</td>
<td>0.479</td>
<td>0.385</td>
<td>0.97</td>
</tr>
</tbody>
</table>

### 4.7.2 $e^*$ in Hardin’s relation

The concept of equivalent void ratio will be used to predict the maximum moduli of granular packing containing fine particles without performing test on mixtures at a macroscopic level. This is one of the advantages of this approach compared to previous methods to predict the stiffness of materials. In this approach, the fitting parameters of Hardin’s relation will be the same as those determined for coarse glass beads. The impact of fines can be expressed through their influence on the equivalent void ratio function (Equation 4.2 and Equation 4.4). Accuracy of the equivalent void ratio concept depends on the parameter $b$ ($f_c$ less than $f_{\text{thr}}$) and $m$ ($f_c$ more than $f_{\text{thr}}$) in the $e^*$ relation. Equation 4.3 was used to predict parameter $b$ and also parameter $m$ in the equivalent void ratio was determined by the back analysis of test data for $G_{\text{max}}$ from the resonant column test.
Effect of fines content

Table 4.4: Summary of fitting parameters for the equivalent void ratio

<table>
<thead>
<tr>
<th>$p'$ [kPa]</th>
<th>$m_{30}$</th>
<th>$m_{40}$</th>
<th>$m_{50}$</th>
<th>$R^2$</th>
<th>RMSD</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.62</td>
<td>0.63</td>
<td>0.63</td>
<td>0.95</td>
<td>5.19</td>
</tr>
<tr>
<td>100</td>
<td>0.59</td>
<td>0.59</td>
<td>0.59</td>
<td>0.96</td>
<td>6.16</td>
</tr>
<tr>
<td>150</td>
<td>0.55</td>
<td>0.56</td>
<td>0.55</td>
<td>0.96</td>
<td>6.68</td>
</tr>
<tr>
<td>200</td>
<td>0.53</td>
<td>0.56</td>
<td>0.56</td>
<td>0.95</td>
<td>8.66</td>
</tr>
<tr>
<td>Ave.</td>
<td>0.573</td>
<td>0.585</td>
<td>0.583</td>
<td>0.96</td>
<td>8.63</td>
</tr>
</tbody>
</table>

Figure 4.30: (a) $G_{\text{max}}$ vs $e^*$ for the mixtures at $p'=200$ kPa; (b) $M_{\text{max}}$ vs $e^*$ for the mixtures at $p'=200$ kPa; (c) $E_{\text{max}}$ vs $e^*$ for the mixtures at $p'=200$ kPa; (d) predicted $G_{\text{max}}$ vs measured $G_{\text{max}}$ for all of the mixtures
Table 4.4 shows the summary of determined values for parameter $m$ in the samples containing 30%, 40% and 50% fines content. The average value of $m$ (Table 4.4) was used to determine the equivalent void ratio ($e^*$) when $f_c$ is more than $f_{thr}$.

Figure 4.30a shows $G_{max}$ vs $e^*$ only for 200 kPa. The solid line in Figure 4.30a is $kf(e)$ curves obtained for coarse glass beads at the same confining pressure as Table 4.3. As can be seen in Figure 4.30a, the data points for all of the mixtures are close to the test data for coarse glass beads. This means the effect of $f_c$ can be captured through the concept of equivalent void ratio for “fines-in-coarse” and “coarse-in-fines” mixtures. Figure 4.30b and Figure 4.30c show $M_{max}$ vs $e^*$ and $E_{max}$ vs $e^*$ only for 200 kPa. The solid lines in these figures are $kf(e)$ curves obtained from $M_{max}$-$e$ and $E_{max}$-$e$ of coarse glass beads at the same confining pressure. Figure 4.30d shows the predicted $G_{max}$ using the Equation 4.5 and by replacing $e$ by $e^*$, versus measured $G_{max}$ using the resonant column device. The results show a good agreement between the predicted and measured results.

4.8 Discussion

The micro-mechanic considerations for sand with fines revealed that the concept of equivalent void ratio, $e^*$, is valid for Hardin’s relation to predict $G_{max}$ of granular materials for a wide range of $f_c$. However, the accuracy of the predicted results depends on the parameters $b$ and $m$ in Equation 4.2 and Equation 4.4 respectively.

![Figure 4.31: The effect of $p'$ on $\lambda$ from calibrations III and V for Jamiolkowski’s $f(e)$](image-url)
The calibration V was the best approach with the lowest RMSD, however λ and m were obtained by back analysis. This model would be preferable if some data for different $f_c$, at least three set of data, were available and so back analysis would be affordable. Calibrations III and IV also require back analysis to a lesser extent. However, the value of λ back analyzed for calibration III showed a good correlation with $p'$ as shown in Figure 4.31, which can be used for the material used in this study. Calibration II, in comparison to other approaches, does not need back analysis and seems to be sufficient to predict $G_{max}$ irrespective of $f_c$. The values of $b$ and $m$ in $e^*$ were determined from the grading properties of sand and fines or from the back analysis of test data. Figure 4.32 shows that $e^*$ in comparison with $e$ provides a unique relationship between $G_{max}$, $f(p')$ and $f(e^*)$ in space, where the fitting parameters of Hardin’s relationship are the same as clean sand. The main advantage of such a single relationship is that if Hardin’s relationship is established for clean sand or sand with any $f_c$, then the relation can be transformed to $e^*$ space where $e$ for clean sand is equal to $e^*$ for sand with fines. This allows us to predict unknown $G_{max}$ for different $f_c$.

Figure 4.32: Maximum shear modulus versus equivalent void ratio and mean effective stress, the meshed surface is: $G_{max} = Af(e^*)f(p')$ for clean Hostun Sand
A detailed description of such a prediction approach for steady state data can be found in Rahman & Lo (2014). This prediction approach is also applicable for $f_c > f_{thr}$ but with the “sand-in-fines” model.

### 4.9 Published data sets

Calibration II for $f_c < f_{thr}$ is evaluated for four collected data sets from the literature as shown in Table 4.5. All the parameters for $e^*$ were obtained from soil grading properties.

<table>
<thead>
<tr>
<th>Sands</th>
<th>Fines</th>
<th>$D_{10}$</th>
<th>$d_{50}$</th>
<th>$f_c$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iruma Z1</td>
<td>Iruma X1</td>
<td>0.13</td>
<td>0.050</td>
<td>0-14</td>
<td>Iwasaki &amp; Tatsuoka (1977)</td>
</tr>
<tr>
<td>Iruma W</td>
<td>Iruma X1</td>
<td>0.65</td>
<td>0.050</td>
<td>0-11</td>
<td>Iwasaki &amp; Tatsuoka (1977)</td>
</tr>
<tr>
<td>Foundary</td>
<td>Sil-co-Sil</td>
<td>0.17</td>
<td>0.01</td>
<td>0-30</td>
<td>Thevanayagam &amp; Liang (2001)</td>
</tr>
<tr>
<td>Volcanic Coarse</td>
<td>same</td>
<td>2.05</td>
<td>0.05</td>
<td>3.2-17.3</td>
<td>Sahaphol &amp; Miura (2005)</td>
</tr>
</tbody>
</table>

Figure 4.33: Measured $G_{max}$ versus predicted $G_{max}$ for published datasets with Jamiołkowski’s void ratio function
A good agreement between predicted and measured $G_{\text{max}}$, with Jamiołkowski’s $f(e)$, was confirmed as shown in Figure 4.33 with $R^2$ of 0.97 and RMSD of 6.95.

### 4.10 Application and limitations

There are soils with almost the same sand grading but different fines along the geological profile, e.g. Christchurch, New Zealand (Green & Cubrinovski 2010) and Ahmedabad, India (Ravishankar et al. 2005), where both sites severely liquefied during earthquakes. In these cases, RC tests on the original material (which may or may not have fines) would be performed to establish Hardin’s relation. This relationship can be transformed to $e^*$ space to predict $G_{\text{max}}$ irrespective of $f_c$. This has a significant advantage when a slight variation in $f_c$ considerably changes $G_{\text{max}}$, an average of 20% change in $G_{\text{max}}$ was observed for a variation in $f_c$ from 0 to 5% as shown in Figure 4.3. This is particularly helpful when a bore log has almost the same host sand but different $f_c$ along the depth profile.

This study has a number of limitations - the concept of $e^*$ was developed based on binary packing which requires sand and fines particle’s disparity ratio to be greater than 6.5. This was achieved in this study for gap graded Huston Sand and quartz powder. The outcomes of this study are not directly transferable to well graded sands. The concept of threshold fines content, $f_{\text{thr}}$, is important for separating “fines-in-sand” and “sand-in-fines” models. However, $f_{\text{thr}}$ is an idealization of a flat transition zone and can not be determined precisely. The $e^*$ at $f_{\text{thr}}$ for “fines-in-sand” and “sand-in-fines” models are close but not exactly the same for the entire range of $e$ used in this study. This is due to the lack of a continuing equation for $e^*$ for $f_c$ before and after $f_{\text{thr}}$. This may have contributed to the scatter of the data points and has already been reflected in the reported RMSDs. The $e^*$ was mostly validated with non-plastic to low plastic fines and thus one may not expect a single Hardin’s relationship for sand with plastic fines. Despite the limitations, the above approach offers a simple and unified methodology to predict $G_{\text{max}}$ irrespective of $f_c$ which has a practical engineering application and may trigger significant research interest in this challenging problem.
Resonant column tests were conducted on clean Hostun Sand to evaluate the influence of mean effective stress, $p'$, and void ratio, $e$, on the maximum shear modulus, $G_{\text{max}}$. Then, the effect of fines on $G_{\text{max}}$ was investigated with a systematic increase in fines content, $f_c$, up to 40%. The major outcomes of the study are:

- A decrease in $G_{\text{max}}$ with an increase in $e$ and an increase in $G_{\text{max}}$ with an increase in $p'$ was observed for clean Hostun Sand. Hardin’s relation was adequate to predict $G_{\text{max}}$ with both Hardin’s and Jamiolkowski’s void ratio function, $f(e)$. A systematic increase in fines content, $f_c$, up to 40% in RC tests showed that $G_{\text{max}}$ decreased with an increase in the $f_c$.

- A micro-CT scan revealed that sand with fines develops two different micro-structures: “fines-in-sand” and “sand-in-fines”. For “fines-in-sand”, fine particles are partially active in sand force structure and for “sand-in fines”, sand particles are floating in fine particles. The void ratio, $e$, for sand with fines does not represent the same force structure as clean sand and is not suitable for a consistent comparison. The equivalent granular void ratio, $e^*$ appears to be a consistent state parameter for density.

- The conversion of $e$ to $e^*$ requires two parameters, $b$ and $m$. $b$ can be obtained from soil grading properties with a fitting parameter $\lambda=0.30$ as in Rahman et al. (2008) and $m$ can be obtained from correlation as in Figure 4.19. However, the parameters, $\lambda$ and $m$ can be further optimized by back analysis. Therefore, five calibration were conducted, including Rahman’s method which does not require back analysis. The best calibration model was obtained when $\lambda$ and $m$ were back analyzed (calibration V). The second best model was obtained when only $\lambda$ was determined by a back analysis of test data (calibration III). The back analyzed $\lambda$ was linearly related to $p'$ which can be used to predict $\lambda$ for Hostun Sand.

- Jamiolkowski’s $f(e)$ provides a better fit than Hardin’s $f(e)$ for a large number of data points with $f_c$. An inverse power function of $e$ appeared to be a better function than a function with a limiting void ratio constant, $x$.

- The unique relationship in space in $G_{\text{max}}$, $\frac{p'}{p_a}$ and $e^*$ cloud be defined for clean sand or sand with fines. The main advantage of such a single relationship is that if Hardin’s relationship is established for clean sand or sand with any $f_c < f_{\text{thr}}$, then
the relationship can be transformed to $e^*$ space where $e$ for clean sand is equal to $e^*$ for sand with fines. This allows us to predict unknown $G_{max}$ for different $f_c$. This prediction approach is also applicable for $f_c > f_{thr}$ but only with the “sand-in-fines” model.

- From the microscopic point of view, the results for glass bead mixtures showed that some fine glass beads were placed in between the gaps in coarse glass beads. They reduced overall $e$, i.e. increased density, but did not contribute to a coarse skeleton. This caused a leftward shift in the trends. However, other fine glass beads may be placed in between the coarse glass beads and may contribute to the coarse skeleton. This reduced the rate of the leftward shift with a higher $f_c$. Therefore, one can observe a decrease in the stiffness of the packing with $f_c$ less than $f_{thr}$. However, for $f_c$ more than $f_{thr}$, fine glass beads dominant in coarse glass beads and, therefore, coarse particles acted like reinforced elements inside fine materials.
5 Stress induced anisotropy

5.1 Introduction

Settlement of soils is one of the challenging problems in the soil-structure interaction during vibration, e.g. earthquake phenomena. Differential settlement may cause distress in the structures founded in the soil mass during earthquake. Maximum shear modulus and modulus degradation curves can be used to estimate the magnitude of ground settlements during earthquake (e.g. Tokimatsu & Seed 1987). The effect of isotropic confining pressure and soil properties on modulus degradation of soils have been studied in the previous works, as referred in Chapter 2. However, soil elements may be subjected to more complicated stress conditions in comparison with the stress conditions applied in the existing studies on intermediate strain properties up to now. Furthermore, the effect of density on maximum shear modulus of sample subjected to stress induced anisotropy at different stress paths must be discussed. Therefore, the objective of this chapter is to find the effect of stress induced anisotropy for different stress paths on the small and especially intermediate strain properties of sand.

This chapter is divided into two main parts. The main goal of part one is to find the effect of stress induced anisotropy on small strain properties of glass bead packing from the microscopic point of view. Therefore, for approaching to this objective, DEM simulation, by means TRUBAL code (Magnanimo et al. 2008), and modified resonant column device were adopted for this study. Since TRUBAL code is restricted to modeling of the spherical particles, the resonant column test were also conducted on spherical glass particles. The outcome of this part is to discuss the effect of stress induced anisotropy for different stress paths on micro and macro mechanical properties of granular packings. In part two, the resonant column test was conducted on Hostun Sand sample and experimental results are presented. The main goal of this section is to find the effect of stress induced anisotropy on small and intermediate strain properties of sand samples. Furthermore, the experimental results on intermediate strain stiffness will be used to modify the empirical
relations for prediction of modulus degradation curves in the samples subjected to stress induced anisotropy.

## 5.2 Glass beads: Numerical and experimental results

### 5.2.1 Overview

In this section, a combined experimental and numerical approach is used to study the shear stiffness at small and intermediate strain in granular materials. A set of experiments is performed on glass beads samples by using a resonant column device under different stress paths, namely isotropic compression, triaxial compression (GB-I) and constant stress ratio $K$ deformation (GB-II). The results revealed that the maximum shear modulus is a function of the stress induced anisotropy, as determined by the specific stress paths. Existing analytical relations fail in describing this extra dependence, either when Equation 2.8 is used or the contribution of (induced) deviatoric stress is included through Equation 2.9. Along with laboratory tests, DEM simulations were performed to propose a micro-mechanical interpretation of the observed behavior. Starting from the numerical observations a modified version of Hardin’s relation is proposed, that includes a dependence of the maximum shear modulus on the coordination number.

![Figure 5.1: Adopted stress paths for performing tests on spherical glass beads](image-url)
The section has been divided into the three parts. In part one, experimental results for samples subjected to isotropic and anisotropic loading will be presented. Afterwards, the experimental results will be presented in comparison with numerical results for calibration and validation of the DEM model. Finally, in part three, the effect of stress induced anisotropy is studied from the micro-mechanical point of view along different stress paths.

5.2.2 Experimental procedure

A series of Resonant column tests were conducted on glass bead samples subjected to stress paths GB-I and GB-II (Figure 5.1). The experimental procedure is the same as presented procedure in Section 3.6. It must be noted that initial void ratio in all of the samples, in experimental and numerical procedure, was equal to 0.57.

5.2.2.1 Experimental results on $G_{max}$

The experimental results on $G_{max}$ for isotropic loading were presented in Section 4.3. Figure 4.9a in Section 4.3 showed the increasing of $G_{max}$ with an increase in the isotropic confining pressure.

Figure 5.2: The effect of stress induced anisotropy on maximum shear modulus of glass bead packing
Figure 5.2 shows the effect of stress induced anisotropy for stress path GB-I and GB-II on maximum shear modulus, $G_{\text{max}}$. As it is apparent from this figure, $G_{\text{max}}$ increases slightly with an increase in the shear stress and it then decreases with further increasing of shear stress for stress path GB-I which is due to increasing of shear contact forces at contact points and decreasing of coordination number, CN. This Figure shows that $G_{\text{max}}$ for stress path GB-II increases with an increase in the shear stress which is due to the increasing of CN and normal contact forces for this stress path. The observed results will be discussed in comparison with numerical results with more details in Section 5.2.4.

5.2.2.2 Experimental results on $G(\gamma)$ and $\eta(\gamma)$

The results of the resonant column test on the dense glass bead samples, subjected to stress induced anisotropy for stress paths GB-I and GB-II, are presented in this section. This section is divided into two main parts: in the first part, the results of isotropic loading are presented and in the second part, the results of stress induced anisotropy are presented.

Figure 5.3: $G$ and $\eta$ versus shear strain for dense glass bead packing with Dr=88% – 91% and subjected to isotropic loading: (a) $G - \gamma$; (b) $\eta - \gamma$
5.2 Glass beads: Numerical and experimental results

**Isotropic loading**

It is well-known that at a given strain amplitude, the shear modulus \( (G) \) and modulus ratio \( (G/G_{\text{max}}) \) increase with an increase in the confining pressure, and damping ratio decreases with an increase in the confining pressure. The observed experimental results (Figure 5.3a and Figure 5.3b) also show the dependency of modulus degradation and damping ratio on the isotropic confining pressure and amplitude of shear strain which is in line with the observed results from DEM simulation and the results from the literature. Figure 5.3a shows that the shear modulus is constant and equal to the maximum shear modulus up to shear strain of 8E-6 which decreases with an increase in the confining pressure. Figure 5.3b shows that damping ratio has the minimum value up to shear strain of 8E-6 which decreases with an increase in the confining pressure. The results show that shear stiffness increases with an increase in the confining pressure and damping ratio decreases with an increase in the confining pressure.

**Anisotropic loading**

Figure 5.4 show the effect of shear strain on the modulus ratio and damping ratio of the glass bead sample with a relative density of 90% and subjected to anisotropic stress conditions at stress paths GB-I and GB-II respectively. Figure 5.4a and Figure 5.4c show that the shear stiffness is constant and equal to the maximum shear modulus up to the shear strain of 6E-6. Figure 5.4b and Figure 5.4d show the effect of stress induced anisotropy on the damping ratio versus the shear strain for stress paths GB-I and GB-II respectively. For stress path GB-I, the experimental test data show that damping ratio decreases slightly with an increase in the vertical stress up to vertical stress of 300 kPa and it then increases at a vertical stress of 350 kPa (Figure 5.4b). However, the damping ratio decreases with an increase in the confining and vertical stress for stress path GB-II (Figure 5.4d). Figure 5.4 shows the effect of stress induced anisotropy on damping ratio for stress path GB-II is more obvious than stress path GB-I. The effect of stress induced anisotropy on shear modulus \( (G) \) is presented in Figure 5.4a and Figure 5.4c for stress paths GB-I and GB-II respectively. For stress path GB-I, Figure 5.4a reveals that the modulus ratio increases slightly with an increase in the vertical stress up to vertical stress of 300 kPa and it then decreases at a vertical stress of 350 kPa. Figure 5.4c shows modulus ratio increases significantly with an increase in the confining pressure and the vertical stress. Figure 5.4 reveals that the effect of stress induced anisotropy on \( G \) for stress path GB-II is more significant than stress path GB-I. The results confirm that \( G \) is affected by stress induced anisotropy but the impact of stress induced anisotropy strongly depends on the adopted stress path and history of loading.
Figure 5.4: $G$ and $\eta$ versus shear strain for dense Glass bead packing, Dr=88% - 91%: (a) $G - \gamma$ curve at stress path GB-I; (b) $\eta - \gamma$ curve at stress path GB-I; (c) $G - \gamma$ curve at stress path GB-II; (d) $\eta - \gamma$ curve at stress path GB-II

5.2.3 Numerical procedure

5.2.3.1 Adopted DEM code

Distinct element method originated from Cundall & Strack (1979) was used to prepare random assemblies of identical, frictional, elastic spheres. 10,000 particles with diameter $d=1.25$mm were randomly generated in a periodic cubic cell. The elastic material properties typical for glass spheres with shear modulus $G_p=29$GPa and Poisson’s ratio, $\nu = 0.2$ were assigned to the particles. The interaction between particles is represented by a non-
central contact force in which the normal component follows the non-linear Hertz’s law and for the tangential component a bilinear relationship with elastic displacement and frictional Coulomb sliding with friction coefficient $\mu_f$ were incorporated (details are given in Makse et al. 1999). For the sake of simplicity gravity was neglected in the numerical experiments. The following definition of the stress tensor was applied to quantify the macroscopic response of a DEM assembly (Cundall & Strack 1979 and Magnanimo et al. 2008):

$$\sigma_{ij} = \frac{1}{V} \sum_{Nc} f^c_i l^c_j,$$

where, $V$ is the total volume of the assembly, $N_c$ is the total number of contacts, $f^c$ denotes the contact force at a contact, and $l^c$ defines the branch vector joining the centers of two contacted particles. The mean effective stress and deviatoric stress can then be determined by $p' = 1/3\sigma_{ii}$ and $q = \sqrt{3s_{ij}s_{ij}/2}$, where $s_{ij} = \sigma_{ij} - \delta_{ij}p'$ ($\delta_{ij}$ is the Kronecker delta). Coordination number was used to characterize the average number of contacts in the sample, that is the particle arrangement. The simple definition of the coordination number is $CN = 2N_c/N_B$, with $N_B$ number of particles in the sample. However, numerical simulation have revealed that at any time during compression, there are some particles with no contacts and some particles with only one contact. None of these particles are contributing to the stable state of stress (Thornton & Antony 1998). Hence a corrected coordination number is introduced:

$$CN = \frac{2N_c - N_1}{(N_B - N_0 - N_1)},$$

where, $N_1$ and $N_0$ are the number of particles with only one or no contacts, respectively. It is worthwhile to notice that the focus is on a unique scalar quantity $CN$ and the orientation of the contact network is neglected.

### 5.2.3.2 Sample preparation

The density of the generated sample in DEM must be close to the experimental value $e \sim 0.57$. It is well-known experimentally that different packing structures are realizable according to the preparation protocol (Figure 5.5). The adopted approach is to generate numerically packings of different structure and determine the relevant variables needed to characterize the elastic response (for details see Magnanimo et al. 2008). Independent on the preparation used, the final structure of the packings which in turn determines the moduli is interest. Here a protocol was employed as suggested by Magnanimo et al. (2008) and Makse et al. (1999), where $\mu$ and $p'$ are set in two different phases during
the compression of the sample, as depicted in Figure 5.5. After random generation in a periodic box, frictionless particles are isotropically compressed from an initial gas to the target density. The compression is stopped just before this density to obtain a dense but non-equilibrated packing. Then, the sample is relaxed, reaching zero-pressure and zero-coordination number. It follows a second isotropic compression with friction to reach the target pressure, $p' = 100\text{KPa}$, as in the experiments. This stress-controlled deformation is carried out using a servo-mechanism that constantly adjusts the applied strain rate according to the difference between the target stress state $\sigma_{ij}^*$ and the measured stress state $\sigma_{ij}$. At each time step, the strain rates $\dot{\epsilon}_{ij}$ are adjusted to the value:

$$\dot{\epsilon}_{ij}^* = \dot{\epsilon}_{ij} + g(\sigma_{ij} - \sigma_{ij}^*),$$

(5.3)

where $g$ is a gain factor that is tuned to achieve equilibrium in an optimal way. In this stage, for a fixed pressure, four different coefficients of friction, $\mu_0 = 0.01, 0.04, 0.06$ and 0.1 were used to create four different packings, respectively. When the coordination number becomes constant with variation of number of cycles the same final friction coefficient $\mu_f = 0.3$ was imposed for all packings and, with further servo-control adjustment, we create an equilibrated condition for the system, until the system is fully equilibrated. All packings are now characterized by the same friction coefficient. However their structure depends on different preparations during the compaction loading. The result is that, for a given pressure and density, packings with different coordination number, ranging from $CN_0 = 5.239$ to $CN_0 = 5.943$ at the highest, have been generated (see Table 5.1). All packings are generated to have the same density $e \simeq 0.57$ and friction coefficient $\mu_f = 0.3$, when they reach the final reference state.

![Figure 5.5: Specimen preparation stages in this study](image-url)
Table 5.1: The effect of $\mu_i$ on coordination number, $CN$, and shear stiffness at isotropic pressure of 200 kPa

<table>
<thead>
<tr>
<th>$p'$</th>
<th>$\mu_i$</th>
<th>$e$</th>
<th>$CN$</th>
<th>$F_N$</th>
<th>$G_{max}[MPa]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.01</td>
<td>0.57</td>
<td>6.04</td>
<td>0.280</td>
<td>249.4</td>
</tr>
<tr>
<td>200</td>
<td>0.04</td>
<td>0.58</td>
<td>5.81</td>
<td>0.294</td>
<td>226.6</td>
</tr>
<tr>
<td>200</td>
<td>0.06</td>
<td>0.58</td>
<td>5.64</td>
<td>0.307</td>
<td>208</td>
</tr>
<tr>
<td>200</td>
<td>0.1</td>
<td>0.58</td>
<td>5.47</td>
<td>0.321</td>
<td>193.7</td>
</tr>
</tbody>
</table>

5.2.3.3 Isotropic compression, triaxial compression and constant $K$ deformation

The four samples described above are subjected to further isotropic compression in order to reproduce the isotropic samples at 50, 100, 200 and 300 kPa as performed in the experiments. During each loading step, the target pressure is maintained with the servo mechanism in Equation 5.3. Starting from the sample at 200kPa a triaxial compression (stress path GB-I) is performed, by applying strain along the axial direction ($e_{11} = e_1$) and keeping the lateral stress, $\sigma_{22} = \sigma_{33} = \sigma_3$, constant through the servo mechanism. Numerical samples with identical ($p', q$) value as applied the experiments (Figure 5.1) are then created. Similarly to the procedure used in experiments, every sample in the triaxial loading path is prepared starting from the initial isotropic state at 200kPa. The goal is to reproduce triaxial loading, so the sample is compressed very slowly ($\gamma_{\partial t}/\Delta_0$ is lower than $5 \times 10^{-3}$, where $\gamma_{\partial t}$ is the accumulated strain in the time step $\partial t$, and $\Delta_0$ the average volumetric compression in the isotropic state). After each increment the system is relaxed until a new equilibrium state is reached. The procedure is repeated for the full set of packings obtained with the different $\mu_i$ preparations, that is for each point on the triaxial path in Figure 5.1, four numerical data are created. Finally, a similar procedure by means of the servo-control mechanism is adopted to deform the samples along a constant stress ratio $K_0 = \sigma_3/\sigma_1$. Configurations that reproduce the stress states in the stress path GB-II are reproduced. Values for the characteristics of the samples are reported in Tables 5.2, 5.3, and 5.4 for isotropic compression, triaxial compression and constant $K$ respectively.
5.2.3.4 Elastic moduli

At each step along the different loading paths (for all four samples) in Figure 5.1 (isotropic, GB-I and GB-II), the maximum shear modulus of the aggregate is calculated by applying an incremental strain to the sample and then allowing it to relax. The corresponding incremental stress response is then measured and modulus is calculated as Magnanimo et al. (2008):

$$G_{12} = \frac{(\sigma_{12})_f - (\sigma_{12})_i}{\epsilon_{12}},$$

where, $(\sigma_{12})_i$ is the shear stress before applying the incremental strain, while $(\sigma_{12})_f$ is the final value after relaxation. The experimental results from the Bochum resonant column device show that the shear stiffness is constant and equal to maximum shear modulus up to the shear strain of less than $5 \times 10^{-6}$. That is, a similar behavior is expected in simulations. For each sample, the procedure is repeated several times by applying increasing amplitudes of the shear strain and the $G_{\text{max}}$ is extracted in the small amplitude range. Figure 5.6 shows the variation of $G_{12}$ with $\epsilon_{12}$ etc.

![Figure 5.6: Shear modulus versus shear strain from experimental and numerical procedure for $p' = 100$ kPa](image)
5.2.4 Numerical results on $G_{\text{max}}$

5.2.4.1 Isotropic loading and calibration of the numerical model

The maximum shear modulus obtained with the four numerical preparations (different $\mu_i$) in the isotropic configurations are reported in Figure 5.7, along with the experimental data. The results show that the maximum shear modulus increases with an increase in the confining pressure. The results from DEM simulations show the significant effect of coordination number on maximum shear modulus in the specimens with the same density ($e=0.57$). The figure shows that, for packings with same pressure and density, a difference in the coordination number due to the preparation, leads to different shear modulus. Three variables (pressure, density and coordination number) are needed to fully characterize the stiffness of an isotropically consolidated sample, in agreement with findings in Magnanimo et al. (2008). Moreover, comparison with experiments shows that samples with less coordinated packings better reproduce our physical samples. The comparison of the macroscopic shear modulus allows for a back analysis. From the back analysis, the internal coordination number of the experimental sample is a value between the values characterized by $CN_0 = 5.09$ and $CN_0 = 5.38$ (created with $\mu_i = 0.01$ and 0.04). Even if these two packings could be used to proceed further in the analysis, in the following the whole set of prepared packings will be used in order to define the boundaries of the physical samples.

5.2.4.2 Anisotropic loading

Results of analyses for stress path GB-I and GB-II are presented in Figure 5.8. Figure 5.8a shows the effect of shear stress, $q$, and coordination number on maximum shear modulus of sample subjected to stress path GB-I. The results show that the maximum shear modulus increases with an increase in the $q$ up to $q$ of 100 kPa and it then decreases significantly with further increase of $q$. The measured results for specimens subjected to stress path GB-II, using DEM simulation, are presented in Figure 5.8b. As can be seen in this figure, the value of $G_{\text{max}}$ increases significantly with an increase in the value of $q$. The experimental results showed that maximum shear modulus was affected by stress induced anisotropy for stress paths GB-I and GB-II. Figure 5.9 shows $G_{\text{max}}$-$q$ curves obtained from experimental method in comparison with the DEM results for the sample with $(\mu_i, \mu_f)=(0.04, 0.3)$. This figure shows the effect of $q$ on maximum shear modulus for stress paths GB-I and GB-II. Figure 5.10 shows the
effect of $q$ on $CN$ for the sample with $(\mu_i, \mu_f) = (0.04, 0.3)$. The results show that $CN$ increases with increasing $q$ for the stress path GB-II. However, for the stress path GB-I, $CN$ increases slightly up to $q$ of 100 kPa and it then decreases with further increasing of $q$.

Figure 5.7: The effect of $p'$ and $CN$ on $G_{max}$ of dense specimen $e = 0.57$: (a) $G_{max}$-$p'$; (b) $G_{max}/f(e)$-$CN$ (isotropic loading)

Figure 5.8: The effect of shear stress, $q$, on maximum shear modulus of dense packing: (a) stress path GB-I; (b) stress path GB-II
Figure 5.9: $G_{max}$ versus $q$ from experimental, filled points, and DEM simulation with $\mu_i=0.04$ and $\mu_f=0.3$, unfilled points

Figure 5.10: $CN$ versus $q$ from DEM simulation with $\mu_i=0.04$ and $\mu_f=0.3$, unfilled points
Hardin’s and Roesler’s relations (Equations 2.8 and 2.9) are two common empirical relations which are applicable to predict $G_{\text{max}}$ in the samples subjected to stress induced anisotropy. Firstly, $G_{\text{max}}$ from the experimental results was normalized with respect to the void ratio (void ratio at the time of RC test). Normalized shear modulus was drawn versus pressure function, $p'^{1/3}$, (Figure 5.11a). Figure 5.11a shows a significant deviation between the results from stress paths GB-I and GB-II in comparison with isotropic loading. As is apparent from this figure, for stress path GB-I, the results up to vertical stress of 300 kPa are in line with the results from isotropic loading but, the scatter of results increases with further increasing of $\sigma_1$. Also, this figure shows that the results for stress path GB-II are not in the line with the results from isotropic loading. Therefore, there is not a unique curve (e.g. Equation 2.8 in Figure 5.11a or Equation 2.9 in Figure 5.11b) to predict the value of $G_{\text{max}}$ in the samples subjected to stress induced anisotropy which will be assessed in the next section from a microscopical point of view.

### 5.2.4.3 Modified Hardin’s relationship

Hardin & Drneich (1972a) developed a well known relationship to predict the maximum shear modulus which was a function of void ratio and pressure. Presented results from DEM simulation show that maximum shear modulus of the specimens with the same density not only depends on the void ratio and confining pressure but also depends on the coordination number, therefore, Hardin’s relation can be rewritten as a function of
5.2 Glass beads: Numerical and experimental results

coordination number, void ratio and pressure (Equation 5.5).

\[ G_{\text{max}} = B f(e) f(CN)(p')^n \]  

(5.5)

where, \( B \) is a constant parameter, \( f(e) \) is void ratio function which is equal to \( e^{-3.98} \) for glass particles with diameter of 1.25 mm (Goudarzy et al. 2014), \( f(CN) \) is the coordination number function, \( p' \) is the isotropic pressure and \( n \) is the pressure exponent. The value of \( n \) was assumed to be \( n = 1/3 \), as can be derived when interaction between particles is the Hertzian contact (Walton 1987, Rothenburg & Bathurst 1989). Coordination number was assumed to be a power function. Equation 5.6 was used to capture the effect of coordination number in Equation 5.5:

\[ f(CN) = (CN - 4)^{n_{cn}} \]  

(5.6)

where, \( CN \) is the coordination number and \( n_{cn} \) is its exponent. In Figure 5.12, the normalized shear modulus, \( G_{\text{max}} \), with respect to the void ratio and pressure \( (G_{\text{max}}/f(e)p^{1/3}) \), was drawn versus the coordination number for all collected packings, created with different procedures and deformed along the three loading paths considered (isotropic, GB-I and GB-II). Surprisingly, all data results were located in a unique curve, irrespective of induced anisotropy or (deviatoric) stress state. The results clearly demonstrate the influence of the micro-structure characterized just by \( CN \). Shear stiffness can be associated with a unique coordination number, that acts as a state variable and able to completely characterize the granular sample, when associated with pressure and volume fraction.

It is worthwhile to notice that the constant parameters of \( B \) and \( n_{cn} \) were also determined by fitting the power function of \( B(CN - 4)^{n_{cn}} \) to the data (Figure 5.12). From the performed regressions the value of \( B \) and \( n_{cn} \) were equal to 2.96 and 0.63 respectively. Therefore, Hardin’s relationship, Equation 5.5, can be used to predict the value of \( G_{\text{max}} \) in samples subjected to anisotropic loading with sufficient accuracy (Figure 5.13a). The accuracy of predicted results can be improved by using the Roesler’s relationship, where, \( f(cn) = (CN - 4)^{0.63} \), \( B = 2.96 \), \( m_1 = 0.12 \) and \( m_3 = 0.21 \) (Figure 5.13b).
Figure 5.12: Normalized results: $G_{\text{max}}/f(e)f(p)$ versus CN for all of the stress paths

Figure 5.13: Comparing the obtained value of $G_{\text{max}}$ from DEM simulation and empirical relation: (a) Hardin’s relation; (b) Roesler’s relation
5.2 Glass beads: Numerical and experimental results

The prepared specimens in section 5.2.3.2 were used to find the effect of shear strain on shear modulus, $G$. To perform the simulations the boundary movement were applied to the specimens slowly. Then, the specimen were relaxed without servo control under the applied shear strain. Afterward, the value of shear stress was extracted from the stress tensor of specimen and the values of shear strain and shear modulus were calculated. For higher amplitude of shear strain the value of boundary movement was increased slowly step by step up to desired shear strain. Figure 5.14 shows the $G - \gamma$ curves from the numerical simulation in comparison with the experimental results for isotropic loading.
The maximum value of $G$ (so-called $G_{\text{max}}$) in these figures are consistent with the obtained values for $G_{\text{max}}$ in Section 5.2.3.2. Figure 5.14 shows that the experimental results are located in the grey band obtained from numerical simulation for the specimens with the same density but with different coordination number. These figures show a good agreement between the results from DEM simulation and the obtained results from RC tests although the method of applying shear strain is different. Figure 5.14c and Figure 5.14d show the effect of coordination number on $G/G_{\text{max}}$. As can be seen in these figures, $G/G_{\text{max}}$ increases slightly with an increase in the coordination number.

### 5.2.6 Interpretation from microscopic point of view

#### 5.2.6.1 Links between macro properties and micro information

Hertz (1882) developed expressions to describe the contact between the smooth elastic surfaces. Hertzian theory has been used as a basic to explain the relationship between shear modulus and confining pressure. According to the Hertz-Mindlin’s theory, the normal and tangential stiffness are function of contact forces and elastic properties of particles (Yimsiri & Soga 2002 and Johnson 1985). The normal contact stiffness ($K_N$) between to identical smooth spheres, is given by:

$$K_N = \frac{2G_p}{1 - \nu_p} \alpha$$

$$\alpha = \left[\frac{3R(1 - \nu_p)}{8G_p}\right]^{1/3} F_N^{1/3}$$

where, $G_p$ is the shear modulus of particles, $\nu_p$ is Poisson’s ratio of particles, $a$ is the radius of contact area, $R$ is the radius of particles and $F_N$ is the normal contact forces.

Deresiewicz (1953) described the tangential contact stiffness ($K_T$) between smooth spheres using Hertzian theory.

$$K_T = \frac{4G_p}{2 - \nu_p} \alpha \left[1 - \frac{F_T}{\mu F_N}\right]^{1/3}$$

where, $\mu$ is the coefficient of friction between particles and $F_T$ is the shear contact force. Chang & Liao (1994) used a micro-mechanic based model to relate the shear modulus, $G_{\text{max}}$, of an assembly of the randomly packed identical spheres to normal, $K_N$, and tangential, $K_T$, stiffness at contact points. Using the static hypothesis which assumes uniform strain and uniform stress respectively, expression for estimate of the elastic modulus was
5.2 Glass beads: Numerical and experimental results

proposed (Otsubo et al. 2015):

\[
G_{\text{max}} = \frac{2NR^2K_N}{3V} \left[ \frac{5(K_T/K_N)}{3 + 2(K_T/K_N)} \right]
\] (5.10)

where, \( N \) is the number of particles and \( V \) is the volume of the sample and \( K_T/K_N \) is equal to \( f(\nu_p) \left[ 1 - \frac{F_N}{\mu F_N} \right]^{1/3} \). Based on Equation 5.10 maximum shear modulus increases with an increase in the coordination number, \( CN \), and normal contact force, \( F_N \), from microscopic level.

Equation 5.10 is helpful for discussion of the observed results at the macroscopic level using the microscopic or contact observations.

5.2.6.2 Polar presentation of contact properties

The outputs of TRUBAL code are a text files including the particles and contact properties. For ease of discussing the results, some analyses were done to present the effect of loading on contact and particle properties graphically. MATHEMATICA programming was used to visualize the results of TRUBAL code. The programming codes were written for 3D presentation. The details of written code for presentation of contact forces and contact distributions are briefly discussed in this section.

If the contact points are drawn, using the output text files, the spherical distribution achieves for normal contact forces (Figure 5.15b). In the next step, the contact force vectors were surrounded by a sphere with unit radius. The unit sphere was divided into the pyramids with the same slopes, \( \theta \). One pyramid was chosen for performing the analyses. The pyramid was defined with a given slope angle (\( \theta \)), then, the boundary surfaces (slant faces) of the pyramid were determined (Figure 5.15a). In the next step, the normal vectors of contacts that were located in this pyramid were determined and the average of them was calculated (Figure 5.15b). In a given pyramid, the average of normal contact forces (\( K \)) was the length of the desired pyramid (Figure 5.15a). Then, the analysis were extended for all of the data to find the length of the other pyramids.

Figure 5.15c shows an example of polar distribution of normal contact forces using discussed procedure for 10000 particles subjected to anisotropic pressure. The same procedure was used for polar presentation of shear contact forces (Figure 5.15d). The unit normal vectors of contacts were used to present the polar distribution of contacts at different time steps. In this case, the length of pyramid (\( K \)) was the number of contacts which were located in the one pyramid.
5.2.6.3 Contact properties

The value of contact properties in respect to the isotropic pressures for dense sample, e=0.58, with the initial and final friction of 0.01 and 0.3 have been summarized in Table 5.2. As can be seen in Table 5.2, the value of coordination number, \( CN \), increases slightly with an increase in the confining pressure although the increasing of \( CN \) with \( p' \) is not so significant in comparison with the other contact parameters. The normal contact force increases significantly with an increase in the isotropic pressure, \( p' \). However, the effect of isotropic pressure on shear contact force and contact distribution is not significant during isotropic loading. As it is apparent from Table 5.2, shear contact force increases with an increase in the isotropic pressure which can be discussed through the friction resistance
and sliding of particles with an increase in the isotropic pressure. Polar distribution of contact properties for isotropic loading are presented in Figure 5.16. As it is apparent from this figure, the normal contact forces increase isotropically with an increase in the confining pressure. Also, this figure shows that contact distribution is isotropic and it remains isotropic with an increase in the confining pressure.

Table 5.2: The effect of isotropic loading on contact properties in the sample with $\mu_i=0.01$ and $\mu_f=0.3$

<table>
<thead>
<tr>
<th>$p'$ [kPa]</th>
<th>$e$ [-]</th>
<th>$CN$ [-]</th>
<th>$F_N$ [-]</th>
<th>$F_T$ [-]</th>
<th>$F_T/F_N$ [-]</th>
<th>$G_{max}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.58</td>
<td>5.88</td>
<td>0.0726</td>
<td>0.0046</td>
<td>0.0629</td>
<td>148.1</td>
</tr>
<tr>
<td>100</td>
<td>0.58</td>
<td>5.97</td>
<td>0.1424</td>
<td>0.0042</td>
<td>0.0292</td>
<td>192.2</td>
</tr>
<tr>
<td>200</td>
<td>0.58</td>
<td>6.04</td>
<td>0.2806</td>
<td>0.0051</td>
<td>0.0181</td>
<td>249.4</td>
</tr>
<tr>
<td>300</td>
<td>0.58</td>
<td>6.08</td>
<td>0.4168</td>
<td>0.0059</td>
<td>0.0143</td>
<td>287.9</td>
</tr>
</tbody>
</table>

Figure 5.16: The effect of isotropic pressure on contact properties of dense glass bead packing, $\mu_i = 0.01$ and $\mu_f = 0.3$
Table 5.3: The effect of stress induced anisotropy on contact parameters in the sample with \( \mu_i=0.01 \) and \( \mu_f=0.3 \), stress path GB-I

<table>
<thead>
<tr>
<th>( \sigma_3 ) [kPa]</th>
<th>( \sigma_1 ) [kPa]</th>
<th>( q ) [kPa]</th>
<th>( e ) [-]</th>
<th>( CN ) [-]</th>
<th>( F_N ) [-]</th>
<th>( F_T ) [-]</th>
<th>( \frac{F_T}{F_N} ) [-]</th>
<th>( G_{max} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>200</td>
<td>0</td>
<td>0.58</td>
<td>6.04</td>
<td>0.2951</td>
<td>0.0123</td>
<td>0.0417</td>
<td>249.4</td>
</tr>
<tr>
<td>200</td>
<td>250</td>
<td>50</td>
<td>0.58</td>
<td>6.03</td>
<td>0.3042</td>
<td>0.0224</td>
<td>0.0735</td>
<td>259.6</td>
</tr>
<tr>
<td>200</td>
<td>300</td>
<td>100</td>
<td>0.58</td>
<td>5.98</td>
<td>0.3297</td>
<td>0.0402</td>
<td>0.1219</td>
<td>265.6</td>
</tr>
<tr>
<td>200</td>
<td>350</td>
<td>150</td>
<td>0.58</td>
<td>5.82</td>
<td>0.3635</td>
<td>0.0549</td>
<td>0.1511</td>
<td>256.3</td>
</tr>
<tr>
<td>200</td>
<td>400</td>
<td>200</td>
<td>0.58</td>
<td>5.53</td>
<td>0.4116</td>
<td>0.0698</td>
<td>0.1695</td>
<td>232.2</td>
</tr>
<tr>
<td>200</td>
<td>420</td>
<td>220</td>
<td>0.58</td>
<td>5.33</td>
<td>0.4407</td>
<td>0.0775</td>
<td>0.1759</td>
<td>201.6</td>
</tr>
</tbody>
</table>

Table 5.4: The effect of stress induced anisotropy on contact parameters in the sample with \( \mu_i=0.01 \) and \( \mu_f=0.3 \), stress path GB-II

<table>
<thead>
<tr>
<th>( \sigma_3 ) [kPa]</th>
<th>( \sigma_1 ) [kPa]</th>
<th>( q ) [kPa]</th>
<th>( e ) [-]</th>
<th>( CN ) [-]</th>
<th>( F_N ) [-]</th>
<th>( F_T ) [-]</th>
<th>( \frac{F_T}{F_N} ) [-]</th>
<th>( G_{max} ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>100</td>
<td>50</td>
<td>0.58</td>
<td>5.07</td>
<td>0.1156</td>
<td>0.0191</td>
<td>0.1651</td>
<td>110.49</td>
</tr>
<tr>
<td>100</td>
<td>200</td>
<td>100</td>
<td>0.58</td>
<td>5.32</td>
<td>0.2157</td>
<td>0.0360</td>
<td>0.1670</td>
<td>166.07</td>
</tr>
<tr>
<td>150</td>
<td>300</td>
<td>150</td>
<td>0.58</td>
<td>5.37</td>
<td>0.3206</td>
<td>0.0538</td>
<td>0.1678</td>
<td>193.40</td>
</tr>
<tr>
<td>200</td>
<td>400</td>
<td>200</td>
<td>0.58</td>
<td>5.53</td>
<td>0.4115</td>
<td>0.0698</td>
<td>0.1695</td>
<td>232.20</td>
</tr>
<tr>
<td>220</td>
<td>440</td>
<td>220</td>
<td>0.58</td>
<td>5.56</td>
<td>0.4509</td>
<td>0.0766</td>
<td>0.1698</td>
<td>240.73</td>
</tr>
<tr>
<td>300</td>
<td>600</td>
<td>300</td>
<td>0.58</td>
<td>5.50</td>
<td>0.6214</td>
<td>0.1047</td>
<td>0.1686</td>
<td>261.44</td>
</tr>
</tbody>
</table>

The obtained values for \( CN \), \( F_N \) and \( F_T \) for stress path GB-I are summarized in Table 5.3. As can be seen in Table 5.3, coordination number and \( F_T/F_N \) are significantly affected by stress induced anisotropy at stress path GB-I. The results show the decreasing of coordination number and increasing of \( F_T/F_N \) with an increase in the vertical stress which have the negative effect on the shear stiffness of packing.

The results of DEM analysis for stress path GB-II are summarized in Table 5.4. As can be seen in Table 5.4, coordination number increases with an increase in the value of \( q \) at this stress path. Furthermore, the results show that the effect of stress induced anisotropy on the value of \( F_T/F_N \) is not significant in comparison with the stress path GB-I.

The effect of stress induced anisotropy on maximum shear modulus can be explained
through the effect of stress state on contact properties which are presented for stress paths GB-I and GB-II in Figure 5.17 and Figure 5.18 respectively.

Figure 5.17 reveals that the normal contact forces in the vertical direction increases significantly with an increase in the vertical stress, $\sigma_1$. However, the value of normal contact forces in horizontal direction was constant and equal to the obtained normal contact forces for specimen subjected to isotropic loading. Also, this figure shows the increasing of shear contact forces ($F_T$) with an increase in the vertical stress. The other micro-mechanical parameter, contact distribution, is also affected with vertical stress. The results show that the contact distribution is oriented to the vertical direction at this stress path.

Figure 5.18 shows the effect of stress induced anisotropy on contact parameters at stress path GB-II. As can be seen in this figure, the normal contact forces increases significantly in all of the directions with an increase in the shear stress, $q$.

Also, Figure 5.18 shows that the shear contact forces ($F_T$) increases with an increase in the vertical stress. The contact distributions are oriented to the vertical direction and has the same orientation for all of the stress states at this stress path.

The experimental and numerical results on glass bead packing revealed that the maximum shear modulus was affected by stress induced anisotropy which was strongly depended on the applied stress paths. For explaining the $G_{\text{max}} - q$ curves, Figure 5.9, the effect of stress induced anisotropy on contact properties are compared in some given stress states, e.g. at $q$ equal to: 50, 100, 150 and 220kPa.

<table>
<thead>
<tr>
<th>$q$ [kPa]</th>
<th>stress path</th>
<th>$CN$ [-]</th>
<th>$F_N$ [-]</th>
<th>$F_T$ [-]</th>
<th>$\frac{F_T}{F_N}$ [-]</th>
<th>$G_{\text{max}}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>GB-I</td>
<td>6.032</td>
<td>0.3042</td>
<td>0.0224</td>
<td>0.0735</td>
<td>259.6</td>
</tr>
<tr>
<td></td>
<td>GB-II</td>
<td>5.0672</td>
<td>0.1156</td>
<td>0.01908</td>
<td>0.1650</td>
<td>110.491</td>
</tr>
<tr>
<td>100</td>
<td>GB-I</td>
<td>5.979</td>
<td>0.329</td>
<td>0.04018</td>
<td>0.12186</td>
<td>265.6</td>
</tr>
<tr>
<td></td>
<td>GB-II</td>
<td>5.324</td>
<td>0.2157</td>
<td>0.03588</td>
<td>0.1663</td>
<td>166.0691</td>
</tr>
<tr>
<td>150</td>
<td>GB-I</td>
<td>5.821</td>
<td>0.3635</td>
<td>0.0549</td>
<td>0.1511</td>
<td>256.3</td>
</tr>
<tr>
<td></td>
<td>GB-II</td>
<td>5.529</td>
<td>0.3208</td>
<td>0.05378</td>
<td>0.1676</td>
<td>193.4</td>
</tr>
<tr>
<td>220</td>
<td>GB-I</td>
<td>5.327</td>
<td>0.4407</td>
<td>0.0775</td>
<td>0.1759</td>
<td>201.06</td>
</tr>
<tr>
<td></td>
<td>GB-II</td>
<td>5.557</td>
<td>0.4511</td>
<td>0.07656</td>
<td>0.1697</td>
<td>240.73</td>
</tr>
</tbody>
</table>
At $q=50$ kPa, in Figure 5.9, the value of $G_{\text{max}}$ at stress path GB-II is less than the obtained value for $G_{\text{max}}$ at the same shear stress for stress path GB-I. The results of DEM simulation for this stress state are summarized in Table 5.5. As can be seen in this table, at the shear stress of 50 kPa the value of coordination number, $CN$, and normal contact force at stress path GB-II are less than stress path GB-I and shear contact force is also more than stress path GB-I, therefore, the stiffness is less than the stress path GB-I at this stress stat. As can be seen in Figure 5.9, for shear stress of 100 and 150 kPa, the value of $G_{\text{max}}$ at stress path GB-II is less than the obtained values for $G_{\text{max}}$ at the same shear stress for stress path GB-I.

<table>
<thead>
<tr>
<th>$q,[\text{kPa}]$</th>
<th>contact distribution</th>
<th>$F_N$</th>
<th>$F_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q=50$</td>
<td><img src="image1" alt="Contact Distribution" /></td>
<td><img src="image2" alt="Contact Distribution" /></td>
<td><img src="image3" alt="Contact Distribution" /></td>
</tr>
<tr>
<td>$q=100$</td>
<td><img src="image4" alt="Contact Distribution" /></td>
<td><img src="image5" alt="Contact Distribution" /></td>
<td><img src="image6" alt="Contact Distribution" /></td>
</tr>
<tr>
<td>$q=150$</td>
<td><img src="image7" alt="Contact Distribution" /></td>
<td><img src="image8" alt="Contact Distribution" /></td>
<td><img src="image9" alt="Contact Distribution" /></td>
</tr>
<tr>
<td>$q=220$</td>
<td><img src="image10" alt="Contact Distribution" /></td>
<td><img src="image11" alt="Contact Distribution" /></td>
<td><img src="image12" alt="Contact Distribution" /></td>
</tr>
</tbody>
</table>

Figure 5.17: Polar presentation of contact properties in specimen subjected to stress path GB-I
At the shear stress of 100 and 150 kPa, the value of $CN$ at stress path GB-I is more than stress path GB-II and also the value of $F_T/F_N$ for the stress path GB-I is less than GB-II, therefore, the value of $G_{max}$ at stress path GB-I is more than obtained value at stress path GB-II.

As can be seen in Figure 5.9, at shear stress of 220 kPa, the value of $G_{max}$ at stress path GB-I is less than the obtained values for $G_{max}$ at the same shear stress at stress path GB-II. At the shear stress of 220 kPa, as can be seen in Table 5.5, the value of $CN$ at stress path GB-II is more than stress path GB-I and also the value of $F_T/F_N$ for the stress path GB-II is less than GB-I, therefore, $G_{max}$ at stress path GB-I is less than $G_{max}$ at stress path GB-II.

<table>
<thead>
<tr>
<th>$q[kPa]$</th>
<th>contact distribution</th>
<th>$F_N$</th>
<th>$F_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q=50$</td>
<td><img src="image1" alt="Contact Distribution" /></td>
<td><img src="image2" alt="FN" /></td>
<td><img src="image3" alt="FT" /></td>
</tr>
<tr>
<td>$q=100$</td>
<td><img src="image4" alt="Contact Distribution" /></td>
<td><img src="image5" alt="FN" /></td>
<td><img src="image6" alt="FT" /></td>
</tr>
<tr>
<td>$q=150$</td>
<td><img src="image7" alt="Contact Distribution" /></td>
<td><img src="image8" alt="FN" /></td>
<td><img src="image9" alt="FT" /></td>
</tr>
<tr>
<td>$q=220$</td>
<td><img src="image10" alt="Contact Distribution" /></td>
<td><img src="image11" alt="FN" /></td>
<td><img src="image12" alt="FT" /></td>
</tr>
</tbody>
</table>

Figure 5.18: Polar presentation of contact properties in specimen subjected to stress path GB-II
5.2.7 Summary

A set of experiments was performed on glass beads samples by using a resonant column device under different stress paths, namely isotropic compression, triaxial compression (GB-I) and constant stress ratio $K$ deformation (GB-II). The results revealed:

- The maximum shear modulus was a function of the stress induced anisotropy, as determined for the specific stress paths. That was the variation of $G_{\text{max}}$ with isotropic/deviatoric stress was not unique but depends on the previous history. Existing analytical relations fail in describing this extra dependence, either when Hardin’s relationship is used or the contribution of (induced) deviatoric stress is included through Roesler’s relation. Along with laboratory tests, DEM simulations were performed and proposed a micro-mechanical interpretation of the peculiar behavior observed.

- It is apparent from experimental results that the maximum shear modulus was significantly affected with stress induced anisotropy at different stress paths. Therefore, DEM analysis were conducted on a granular packing to find the effect of stress induced anisotropy on contact properties for various stress paths. From macroscopic point of view, the maximum shear stiffness of granular packing increased with an increase in the mean effective stress which was due to the increasing of the normal contact forces at contact points from the microscopic point of view.

- DEM results revealed that Coordination Number, $CN$, is affected significantly by sample preparation. Starting from the numerical observations a modified version of Hardin’s relationship is proposed, that included a dependence of the maximum shear modulus on the coordination number.

- The results show that the effect of stress induced anisotropy on maximum shear modulus strongly depends on the applied stress path. The experimental and numerical results revealed that at the same $q$, maximum shear modulus is different in the specimens which are consolidated at stress paths GB-I, GB-II. Micro-mechanical observations show that the increasing of $F_T/F_N$ and decreasing of coordination number in stress path GB-I is faster than the other stress path. Therefore, with reference to Equation 5.10, the stiffness decreases with an increase in the vertical stress in this stress path. Furthermore, the DEM results show that the coordination number, $CN$, and normal contact forces, $F_N$, increase with an increase in the value of $\sigma_1$ at
stress path GB-II and isotropic stress path. Therefore, stiffness increases with an increase in the $\sigma_1$ at these stress paths in comparison with stress path GB-I.

- The results at macroscopic level reveal that Roesler’s relationship with coordination number function can be used with sufficient accuracy to predict the maximum shear modulus in the specimens subjected to anisotropic loading.
5.3 Hostun sand: Experimental results and empirical relationships

5.3.1 overview

A series of stress induced anisotropic tests were conducted on dry Hostun sand with a relative density of 35%-90% to evaluate the effect of the stress path on small and intermediate strain properties ($G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$). From the experimental study using true triaxial device, Sadek (2006) concluded that the effect of the direction of sand pluviation on maximum shear modulus of Hostun sand is not significant. This means that the effect of pluviation of Hostun sand on the fabric of sample is not significant. Therefore, in this section, all of the samples were prepared by a dry air pluviation method (Section 3.4). Stress induced anisotropy was applied to the samples by different stress paths (Figure 3.14a). The experimental procedure was explained in section 3.6. The main results of this experimental program are presented in this Section. For ease of discussion, this section is divided into three main parts. In part one, the effect of stress induced anisotropy on the small strain properties ($G_{\text{max}}$) of dry Hostun Sand is presented. The effect of stress induced anisotropy on intermediate strain properties ($G(\gamma)$ and $\eta(\gamma)$) of dry Hostun Sand for different stress paths will be discussed in part two. In part three, the effect of stress induced anisotropy on the reference shear strain, $\gamma_r$, (Equation 2.10) will be presented. These results are used to develop an empirical relationship to predict the value of $G/G_{\text{max}}$ of samples subjected to stress induced anisotropy.

5.3.2 Test results on $G_{\text{max}}$

The effects of void ratio and stress induced anisotropy on the maximum shear modulus ($G_{\text{max}}$) of dry Hostun Sand are explained in this section. Adopted stress paths are the same as Figure 3.14a.

- Stress path I

The experimental results for this stress path are presented in Figure 5.19. The test data in Figure 5.19a reveal that the value of $G_{\text{max}}$ increases with an increase in the shear stress, $q$, for this stress path. Also, the significant effect of confining pressure on the maximum shear modulus can be concluded from Figure 5.19a.

Figure 5.19b reveals that $G_{\text{max}}$ decreases with an increase in the void ratio, $e$. The experimental test data for stress path I show that the density of sample not only changes
the value of $G_{\text{max}}$ but can also have an effect on the trend of the $G_{\text{max}} - q$ curve in comparison with the dense sample. As can be seen in Figure 5.19b, the maximum shear modulus, $G_{\text{max}}$, increases with an increase in $q$ up to $q$ of 300 kPa, but the slope of the $G_{\text{max}} - q$ curve of the loose sample ($e=0.91$) is different to the dense sample ($e=0.71$) for $q$ of more than 300 kPa. $G_{\text{max}}$ was normalized with respect to the $G_{\text{max}}$ obtained for isotropic loading. This normalization is helpful to show the effect of density on the trend of the observed results.

Figure 5.19: $G_{\text{max}} - q$ curves for stress path I: (a) the effect of $\sigma_3$, $e = 0.73$; (b) the effect of the void ratio

Figure 5.20: $\frac{G_{\text{max}}}{G_{\text{max}}(\text{isotropic})} - q$ curves for stress path I: (a) the effect of $\sigma_3$, $e = 0.73$; (b) the effect of the void ratio
Figure 5.20a shows normalized $G_{\text{max}}$ versus $q$ in the samples with a void ratio of 0.73. This figure shows that the effect of isotropic confining pressure on the trend of normalized $G_{\text{max}}$ versus $q$ is not significant. However, Figure 5.20b shows the significant effect of density on results which is due to the effect of anisotropic loading on the fabric of the samples. For instance, during anisotropic loading, due to the compaction of loose sample, the value of the coordination number increases faster and, consequently, the maximum shear stiffness increases faster than the dense sample.

- **Stress path II**

The experimental test results for stress path II are presented in Figure 5.21. As it is apparent from Figure 5.21a, the value of $G_{\text{max}}$ increases slightly with an increase in the $q$ and it then decreases significantly with a further increase in $q$ ($q > 0.7p'$). This figure also shows that the value of $G_{\text{max}}$ is significantly affected by $p'$. The test data show that the value of $G_{\text{max}}$ increases with an increase in the value of $p'$ (Figure 5.21a). Furthermore, the test results show that the value of $G_{\text{max}}$ decreases with an increase in the void ratio (Figure 5.21b) although the trend of results for the loose and dense samples is obviously different, especially for $q > 0.7p'$ for this stress path. This difference is due to the change of coordination number and void ratio, during the compaction of the sample, which has a direct effect on the value of $G_{\text{max}}$.

Figure 5.21: $G_{\text{max}} - q$ curves for stress path II: (a) the effect of $p'$, $e = 0.73$; (b) the effect of the void ratio ($K=0.5$)
5.3 Hostun sand: Experimental results and empirical relationships

Figure 5.22: $G_{\text{max}} - q$ curves for stress path III: (a) the effect of $K = \sigma_h/\sigma_v$, $e = 0.71$; (b) the effect of the void ratio

Figure 5.23: $G_{\text{max}}$ versus $q$ for Hostun Sand with $e_0 = 0.72$ and stress paths I, II and III

- Stress path III

Figure 5.22 presents the effect of stress induced anisotropy on the maximum shear modulus for stress path III. As is apparent from Figure 5.24a, the value of $G_{\text{max}}$ increases significantly with an increase in the value of $q$ for this stress path. This figure shows that
the value of $G_{\text{max}}$ increases with an increase in the stress ratio, $K = \sigma_h/\sigma_v$. Furthermore, the experimental results at stress path III show that the value of $G_{\text{max}}$ decreases with an increase in the void ratio (Figure 5.22b).

For example, Figure 5.23 shows the effect of stress induced anisotropy at different stress paths on the maximum shear modulus of Hostun Sand with a relative density of 86%. For stress path I, this figure shows that the maximum shear modulus, $G_{\text{max}}$, increases with an increase in the value of $q$ but the rate of increase in $G_{\text{max}}$ decreases significantly with $q$ of more than 200 kPa. For stress path II, $G_{\text{max}}$ increases slightly with an increase in the value of $q$, but $G_{\text{max}}$ decreases significantly with $q$ of more than 150 kPa ($q > 0.7\rho'$) (Figure 5.23). For stress path III, experimental results reveal that $G_{\text{max}}$ increases as long as $q$ increases (Figure 5.23).

5.3.2.1 Empirical relationship for $G_{\text{max}}$

Hardin’s relation (Equation 2.8) and Roesler’s relation (Equation 2.9) were used to predict the maximum shear modulus in the soil samples subjected to stress induced anisotropy. Hardin’s void ratio function (Equation 4.6) was also used to capture the effect of the void ratio in these relationships. As discussed in Chapter 2, Hardin believed that $G_{\text{max}}$ is independent of shear stress. Thus, Equation 2.8 with sufficient accuracy can be used to predict the value of $G_{\text{max}}$ but the average of stresses ($\sigma_1+\sigma_3$) must be used instead of $p'$. According to the performed regressions, constant parameters $A$ and $n$ in Hardin’s relation (Equation 2.8) were equal to 0.83 and 0.44 respectively (Chapter 4). Roesler’s relation (Equation 2.9) includes four constant parameters: $x$ in void ratio function, $A$ constant parameters of Equation 2.9 and stress exponents ($n_1$ and $n_3$). Parameters $x$ and $A$ in Roesler’s relation were assumed to be the same as the values obtained for Hardin’s relation. However, the unknown values of $n_v$ and $n_h$ were determined by regression of the test results for stress path I to get the best value for $R^2$ (Figure 5.24). Then, the average of the obtained values for the stress exponents, $n_3$ and $n_1$, were used to predict the value of $G_{\text{max}}$ for stress paths II and III.

The test data for stress path I were used for determining the value of stress exponents in Roesler’s relation. $G_{\text{max}}$ was normalized with respect to the $Af(e)(\sigma_3^{n_3})$, where A and f(e) were the same as the determined values for Hardin’s relation and $n_3$ was assumed to be a value less than 0.5. Normalized shear modulus were drawn versus vertical stress, $\sigma_1$. Figure 5.24a shows an example of normalized shear modulus versus $\sigma_1$. The value of $n_h$ was changed to get a value close to 1 for $K$ factor in Figure 5.24a.
Figure 5.24: Fitted curves to the experimental data for stress paths I: (a) normalized $G_{\text{max}} - \sigma_v$ curve; (b) $G_{\text{max}} - \sigma_v$ curves for samples with various densities

This procedure was performed for all of the samples subjected to stress induced anisotropy at stress path I. Figure 5.24b shows the test data for samples with various densities, the solid lines fitted are curves predicted with Roesler’s relation. The average value of $n_1$ and $n_3$ were used as stress exponents in Roesler’s relation. The average value of the stress exponents, $n_3$ and $n_1$, in Roesler’s relationship (Equation 2.9), were equal to 0.23 and 0.21 respectively which confirms that the effect of confining pressure on the maximum shear modulus is more than the effect of vertical stress.

Figure 5.25a and Figure 5.25b show the value of $G_{\text{max}}$, predicted with Equation 2.8 and Equation 2.9, in comparison with the measured values for stress paths I, II and III. Figure 5.25a shows that both of the relations (Equation 2.8 and Equation 2.9) predict the value of $G_{\text{max}}$ with the same accuracy up to the vertical stress of 400 kPa for stress path I and 300 kPa for stress path II. However, at a vertical stress of more than 400 kPa, in a dense sample volumetric changes in the sample was not significant during RC tests, therefore, the rate of increase in the maximum shear modulus was affected by contact forces at a microscopic level (contact force ratio). The impact of shear force can be observed through Equation 2.8 in comparison with Equation 2.9. As can be seen in Figure 5.25a, the accuracy of the predicted $G_{\text{max}}$ using Equation 2.9 is more than Equation 2.8, whereas Equation 2.8 is independent of shear stress. In the loose sample due to the compaction of the sample, the coordination number increases which has a direct effect on $G_{\text{max}}$. Therefore, in the loose sample the value of $G_{\text{max}}$ increases with an increase in the value of $q$ at stress path I (Figure 5.25a). The same behaviour was observed for samples subjected to stress path II. Figure 5.25b shows an example of measured maximum shear modulus for stress path...
III. The results for stress path III (Figure 5.25b) show that Equation 2.8 and Equation 2.9 can be used to predict the value of $G_{max}$ with the same accuracy. Figure 5.26 shows the accuracy of the predicted $G_{max}$ using Equation 2.8 and Equation 2.9 in comparison with the measured values for stress paths I, II and III. Figure 5.26a shows the predicted $G_{max}$ using Equation 2.8 versus measured $G_{max}$ for dry Hostun Sand with a different void ratio.

![Graph](image1)

(a)

![Graph](image2)

(b)

Figure 5.25: The predicted and measured $G_{max}$ for dry Hostun Sand subjected to: (a) stress paths I and II; (b) stress path III
Figure 5.26b shows the predicted results using Equation 2.9 versus the experimental results. A comparison of Figure 5.26a and Figure 5.26b reveals that the predicted results using Equation 2.9 have a higher accuracy in comparison with the predicted results using Equation 2.8 for stress path I. The same conclusion can be extracted for the other stress paths. However, as can be seen in Figure 5.26e and Figure 5.26f, both of the relationships, Equation 2.8 and Equation 2.9, predict the maximum shear modulus with the same accuracy for stress path III.
Figure 1: $G_{\text{max}}$ versus void ratio at $p' = 200$ kPa for: (a) Clean Hostun sand (b) Clean Hostun sand containing 30% fine content.

Figure 5.26: The accuracy of predicted $G_{\text{max}}$ with empirical relationships and determined fitting parameters based on the test data for stress path I: (a) Hardin’s relationship for stress path I; (b) Roesler’s relation for stress path I; (c) Hardin’s relation for stress path II; (d) Roesler’s relation for stress path II; (e) Hardin’s relation at stress path III; (f) Roesler’s relation for stress path III.
5.3.3 Published data sets on $G_{\text{max}}$

The experimental results for $G_{\text{max}}$ obtained from the resonant column device have been compared with previous studies in Figure 5.27. It must be noted that the published results for stress path I are presented in this Figure. As can be seen in this figure, the effect of stress induced anisotropy on dense samples for stress path I depends on the stress ratio. Stiffness increases with an increase in vertical stress up to stress ratio 2 and it then decreases with a further increase in vertical stress for this stress path which is in agreement with the observed experimental results in this study.

5.3.4 Test results on $G(\gamma)$

The resonant column tests were conducted on the samples which were consolidated at stress paths I, II and III to assess the effects of stress path and stress induced anisotropy on the curves of damping ratio and modulus ratio. Figure 5.28 shows the effect of shear strain on the shear modulus of clean Hostun sand with a relative density of 86% – 90% which were subjected to isotropic and anisotropic stress conditions at stress paths I, II and III respectively.

![Figure 5.27: Normalized $G_{\text{max}}$ versus $\sigma_v/\sigma_h$ from the literatures for stress path I in comparison with this study for the same stress path](image-url)
Figure 5.28: $G$ versus shear strain for clean Hostun Sand, Dr=72% – 75%: (a) isotropic loading; (b) anisotropic loading at stress path I; (c) anisotropic loading at stress path II; (d) anisotropic loading at stress path III

Figure 5.28 shows that the shear stiffness is constant and equal to the maximum shear modulus up to the shear strain of 1E-5. Figure 5.28b shows that the shear stiffness increases with an increase in the vertical stress ($\sigma_1$) up to the vertical stress of 600 kPa. However, for stress path II (Figure 5.28c), the shear stiffness increases slightly with an increase in the shear stress, up to the shear stress of less than $0.7\sigma'$, and it then decreases with further increases in the shear stress. The results for stress path III (Figure 5.28d) show the shear stiffness increases with an increase in the confining pressure at this stress path. Additionally, the presented results show the dependency of shear stiffness on the shear strain in the samples subjected to the anisotropic loading for stress paths I, II and
5.3 Hostun sand: Experimental results and empirical relationships

Figure 5.29 shows the effect of stress induced anisotropy on the damping ratio versus the shear strain for isotropic and anisotropic loading at stress paths I, II and III respectively. For isotropic loading, the experimental test data show that damping ratio decreases significantly with an increase in the confining pressure (Figure 5.29a). Figure 5.29b shows the effect of stress induced anisotropy on damping ratio for stress path I. The results show that damping ratio decreases slightly with an increase in the vertical stress. Figure 5.29c shows that the effect of shear stress on damping ratio for stress path II.

![Graph for isotropic loading](image1)

![Graph for anisotropic loading at stress path I](image2)

![Graph for anisotropic loading at stress path II](image3)

![Graph for anisotropic loading at stress path III](image4)

Figure 5.29: Damping ratio, $\eta$, versus shear strain for clean Hostun Sand, Dr=72%–75%: (a) isotropic loading; (b) anisotropic loading at stress path I; (c) anisotropic loading at stress path II; (d) anisotropic loading at stress path III.
The results reveal that damping ratio decreases slightly with an increase in the shear stress up to shear stress of 40 kPa and it then increases with further increase in the shear stress (Figure 5.29c). For stress path III, however, the damping ratio decreases with an increase in the confining and vertical stress (Figure 5.29d). As it is apparent from Figure 5.29, the effect of stress induced anisotropy on damping ratio for isotropic loading and stress path III are more obvious than stress paths I and II. The relationship between $G/G_{\text{max}} - \log \gamma$ curves for dry Hostun sand subjected to isotropy and stress induced anisotropy are presented in Figure 5.30. For isotropic loading, the experimental test data show that $G/G_{\text{max}}$ increases significantly with an increase in the confining pressure (Figure 5.29a).
For stress path I, the experimental test data show that $G/G_{\text{max}} - \log\gamma$ curve was slightly shifted to higher values in comparison with the results for isotropic loading at the confining pressure of 200 kPa (Figure 5.30b). Since the test data are close to each other the effect of stress induced anisotropy on $G/G_{\text{max}} - \log\gamma$ curves is not so obvious in Figure 5.30b. Figure 5.30c shows the effect of stress induced anisotropy on $G/G_{\text{max}} - \log\gamma$ curve for stress path II. The results reveal that $G/G_{\text{max}}$ increases slightly with an increase in the shear stress up to shear stress of 40 kPa and it then decreases with further increase in the shear stress (Figure 5.30c). For stress path III, however, the $G/G_{\text{max}}$ increases with an increase in the confining and vertical stress (Figure 5.30d). Figure 5.30 also shows the
effect of stress induced anisotropy on $G/G_{\text{max}}$ for isotropic loading and stress path III are more obvious than stress paths I and II. Figure 5.31 shows the variation of $\gamma_r$ in respect to $\sigma_1$ for all of the applied stress paths.

Figure 5.31 shows that the value of $\gamma_r$ increases significantly with an increase in the confining pressure. This figure shows that the value of $\gamma_r$ increases slightly with an increase in the vertical stress in the samples subjected to anisotropic loading at stress path I. As can be seen in Figure 5.31, at stress path II, the value of $\gamma_r$ increases slightly with an increase in the shear stress up to shear stress of less than 40 kPa and it then decreases significantly with an increase in the shear stress. The value of $\gamma_r$ increases with an increase in the confining pressure for the sample subjected to stress path III (Figure 5.31).

![Figure 5.31: The effect of stress induced anisotropy on reference shear strain](image)

**5.3.4.1 Empirical relationship**

Equation 2.13 and Equation 2.14 are not valid to determine the value of $\gamma_r$ for samples subjected to the anisotropic loading at stress path I (Tatsuoka et al. 1979). Therefore, the back analysis of the test data, using Equation 2.10, was used to determine the value of $\gamma_r$ from the experimental test data for sample subjected to the isotropic and anisotropic loading.

Equation 2.15 was used to predict the value of $\gamma_r$ in the samples subjected to anisotropic loading. This relation is based on the mean effective stress ($p'$). However, the experimental
test data show that the value of $\gamma_r$ is affected by shear stress at various stress paths (Figure 5.31). Therefore, in this study, Equation 2.15 was modified as a function of principle stress components instead of mean effective stress (Equation 5.11):

$$\gamma_r = \gamma_{r1} \left( \frac{\sigma'_1}{p_a} \right)^{m_1} \left( \frac{\sigma'_3}{p_a} \right)^{m_3}$$

(5.11)

where, $\gamma_r$ is the reference shear strain, $\gamma_{r1}$ is the reference shear strain for isotropic loading at $p' = 100$ kPa, $\sigma_1$ and $\sigma_3$ are the maximum and minimum principle stress components respectively, and $m_1$ and $m_3$ are exponents of vertical and horizontal stresses respectively. Equation 5.11 was used to predict the value of $\gamma_r$ for samples subjected to isotropic and anisotropic loading at different stress paths. $\gamma_{r1}$ is the value of $\gamma_r$ for a certain isotropic pressure ($p' = 100$ kPa). Afterwards, Equation 5.11 was fitted to the data for stress path I in Figure 5.31 to find the parameters $m_1$ and $m_3$ (solid line in Figure 5.31). By performing this regression, the values of 0.08 and 0.31 were obtained for $m_1$ and $m_3$ respectively. Determined fitting parameters ($m_1$ and $m_3$) for stress path I were used in Equation 5.11 to predict the value of $\gamma_r$ for a sample subjected to the other stress paths (dashed lines, in Figure 5.31). As can be seen in Figure 5.31, the results show a good agreement between the predicted and measured values.

### 5.3.4.2 Prediction of $G/G_{\text{max}}$

The normalized shear strain, $\gamma/\gamma_r$, is the key parameter to predict the $G/G_{\text{max}}$ using Equation 2.10 and Equation 2.12. To predict the $G/G_{\text{max}}$ for the sample subjected to anisotropic loading, the shear strain was normalized with respect to the $\gamma_r$ which was obtained from Equation 5.11. Figure 5.32 shows the $G/G_{\text{max}}$ curves versus the normalized shear strain for the samples subjected to the isotropic (Figure 5.32a) and anisotropic loading (Figure 5.32b, Figure 5.32c and Figure 5.32d). In Figure 5.32, the solid lines are the predicted results using Equation 2.12, where the fitting parameters $a$ and $b$ were equal to 0.05 and 1, and $\gamma_r$ was determined with Equation 5.11. Figure 5.33 shows the damping ratio versus the normalized shear strain for all of the stress paths (Isotropic (Figure 5.33a), anisotropic loading (Figure 5.33b, Figure 5.33c and Figure 5.33d)). The predicted curves, using Equation 2.16, have been added as a solid lines in these figures. The values of $\eta_{\text{min}}$, $c_1$ and $c_2$ in Equation 2.16 were equal to 0.0102, 0.393 and 0.808 respectively. The results show a good agreement between predicted and measured results.
Figure 5.32: \( G/G_{\text{max}} \) versus normalized shear strain of clean Hostun Sand with \( Dr=72\% - 75\% \) and subjected to: (a) isotropic loading; (b) anisotropic loading at stress path I; (c) anisotropic loading at stress path II; (d) anisotropic loading at stress path III
Figure 5.33: Damping ratio, $\eta$, versus normalized shear strain for clean Hostun Sand with Dr=72% - 75% and subjected to: (a) isotropic loading; (b) anisotropic loading at stress path I; (c) anisotropic loading at stress path II; (d) anisotropic loading at stress path III
5.3.5 Summary

The stress induced anisotropic tests were conducted on Hostun sand specimens. In this experimental study, Hostun Sand was subjected to four main stress paths: isotropic, stress path I, stress path II and stress path III. For isotropic loading confining pressure was increased from 55 kPa to 200 kPa. For stress path I, confining pressure ($\sigma_3$) was kept constant at 200 kPa and vertical stress was increased. For stress path II, $p'$ was kept constant at 200 kPa and $q$ was increased and finally, for stress path III, confining pressure was increased to 100, 200 and 300 kPa then, the vertical stress was increased up to $\sigma_1 = 2\sigma_3$. After that, the resonant column tests were conducted on the samples. Measured results for induced anisotropy tests on dry Hostun sand under three different stress paths were presented in this work. Experimental results showed that the impact of stress induced anisotropic on small and intermediate strain properties of dry Hostun sand ($G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$) depends on the adopted stress path. In stress path I, the effect of stress ratio was significant on $G_{\text{max}}$ but the influence of stress ratio on $G/G_{\text{max}}$ and damping ratio was not significant. At stress path II and III, $G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$ were significantly affected by stress induced anisotropy. The experimental results on Hostun sand samples showed the significant effect of density on maximum shear modulus of sample subjected to stress induced anisotropy although the effect of density on $G/G_{\text{max}}$ and $\eta$ was not significant.
6 Conclusions and recommendations

6.1 The effect of fines content

Resonant column tests were conducted on clean Hostun Sand to evaluate the influence of confining stress, $p'$, and void ratio, $e$, on $G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$. Then the effect of fines on $G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$ was investigated with a systematic increase in fines content, $f_c$ up to 40%. The major outcomes of the study are:

- A decrease in $G_{\text{max}}$ with an increase in $e$ and an increase in $G_{\text{max}}$ with an increase in $p'$ was observed for clean Hostun Sand. Hardin’s relation is adequate to predict $G_{\text{max}}$ with both Hardin’s and Jamiolkowski’s void ratio function, $f(e)$. A systematic increase in fines content, $f_c$ up to 40% in RC tests showed that $G_{\text{max}}$ decreased with an increase in $f_c$ which is consistent with earlier studies for sand with non-plastic fines.

- A micro-structural study with a micro CT scan revealed that sand with fines develops two different micro-structures: “fines-in-sand” and “sand-in-fines”. For “fines-in-sand”, fine particles were partially active in the sand force structure and for “sand-in-fines”, sand particles were floating in fines particles. The void ratio, $e$ did not represent the same force structure as clean sand and is not suitable for a consistent comparison. The equivalent granular void ratio, $e^*$, appears to be a consistent state parameter for density.

- The conversion of $e$ to $e^*$ requires two parameters, $b$ and $m$. $b$ can be obtained from soil grading properties with a fitting parameter $\lambda=0.30$ as in Rahman et al. (2008) and $m$ can be obtained from correlation as in Figure 4.19. However, the parameters, $\lambda$ and $m$ can be further optimized by back analysis. Therefore, five regression analyses were conducted, including Rahman’s method that does not require back analysis. The best regression model was obtained when $\lambda$ and $m$ were back analyzed (regression V). The second best model was obtained when only $\lambda$ was determined by back analysis of test data (regression III). The back analyzed $\lambda$ was linearly related
to $p'$ which can be used to predict $\lambda$ for Hostun sand.  
To predict $G_{\text{max}}$ as a function of $e^*$ and $p'$, Rahman’s relation with the same proposed fitting parameters (regression I) can be used when $f_c$ is less than $f_{\text{thr}}$ and the $m$ value is used for $f_c$ greater than $f_{\text{thr}}$ (regression II and regression III).

- Jamiolkowski’s $f(e)$ gives a better fit than Hardin’s $f(e)$ for a large number of data points with $f_c$. An inverse power function of $e$ appeared to be a better function than a function with a limiting void ratio constant, $x$.

These conclusions are derived from the test data for gap graded sand with non-plastic fines and may not be applicable to sand with plastic fines. Furthermore, a series of tests was conducted on glass bead mixtures containing 0, 10, 20, 30, 40 and 50 percentage fine glass beads using the resonant column test and piezoelectric elements. The outcomes of the study are:

- $G_{\text{max}}$, $M_{\text{max}}$ and $E_{\text{max}}$ in glass bead mixtures decrease with an increase in fines content up to 30% $f_c$ and then decrease with further increase in $f_c$.

- The conversion of $e$ to $e^*$ requires two parameters, $b$ and $m$. $b$ was obtained from particle size properties with a fitting parameter $\lambda=0.30$ as in Rahman et al. (2008) and $m$ was optimized by back analysis of test data for $G_{\text{max}}$. The results confirmed the applicability of $e^*$ in Hardin’s relation to predict the value of $G_{\text{max}}$, $M_{\text{max}}$ and $E_{\text{max}}$.

- Micro-CT imaging shows that the number of contacts between coarse particles decreases with an increase in the fines content and therefore, stiffness decreases with fines content.

### 6.2 Stress induced anisotropy

The small and intermediate strain properties of dry Hostun Sand subjected to three different stress paths were presented in this paper. Experimental results show that the impact of stress induced anisotropic on the small and intermediate strain properties of dry Hostun sand ($G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$) depends on the adopted stress path. In stress path I, the effect of the stress ratio on $G_{\text{max}}$ was significant but the influence of the stress ratio on $G/G_{\text{max}}$ and damping ratio was not significant. At stress paths II and III, $G_{\text{max}}$, $G(\gamma)$ and $\eta(\gamma)$ were significantly affected by stress induced anisotropy. The experimental
results on Hostun Sand samples show a significant effect of density on the maximum shear modulus of the sample subjected to stress induced anisotropy although the effect of density on $G/G_{\text{max}}$ and $\eta$ was not significant. The results show that the Stokoe relationship, Equation 2.15, can be used in terms of stress components, Equation 5.11, to predict the value of $\gamma_r$ in the soil samples subjected to stress induced anisotropy.

### 6.3 DEM simulations

It is apparent from the experimental and numerical results that the maximum shear modulus was significantly affected by stress induced anisotropy at different stress paths. Therefore, DEM analyses were conducted on granular packings to find the effect of stress induced anisotropy on contact properties at various stress paths. From a macroscopic point of view, the maximum shear stiffness of granular packing increased with an increase in the confining pressure which was due to the increase in the normal contact forces at contact points from the microscopic point of view. Furthermore, the DEM results reveal that Coordination Number, $CN$, was affected significantly by sample preparation. From the performed regressions, the effect of the coordination number on the maximum shear modulus was captured through the coordination number function in Hardin’s relation. The results show that the effect of stress induced anisotropy on the maximum shear modulus strongly depends on the applied stress path. The experimental and numerical results reveal that for constant $q$, the maximum shear modulus was different in the specimens which were consolidated at isotropic and anisotropic loading at stress paths GB-I and GB-II. Micro-mechanical observations show that the increase in $F_T/F_N$ and decrease in the coordination number for stress path GB-I was faster than the other stress paths. Therefore, with reference to Equation 5.10, stiffness decreases with an increase in the shear stress in this stress path. Furthermore, the DEM results show that the coordination number, $CN$, and normal contact forces ($F_N$) increase with an increase in the value of $q$ at stress path GB-II and the isotropic stress path. Therefore, stiffness increases with an increase in the $\sigma_v$ at these stress paths in comparison with stress path GB-I. Additionally, the results at the macroscopic level reveal that Roesler’s relation with the coordination number function can be used with sufficient accuracy to predict the maximum shear modulus in the specimens subjected to anisotropic loading.
6.4 Suggested future works

This thesis could be developed further in the future. Based on the findings of this study further research is suggested as follows:

- Most of the previous studies have been conducted on the fine in sand mixtures. However, the studies on sand in fine mixtures are rare and additional studies are essential to determine the effects of particle characteristics, e.g. size and shape of fine particles, on the small and intermediate strain properties of granular materials.

- There is no relationship to predict the value of $m$ in $e^*$ for the sand in fine mixture. Therefore, additional systematic studies on the mixtures containing fines content beyond the $f_{thr}$ must be conducted to determine the empirical relation between the parameter $m$ and the particle characteristics of fine and coarse particles.
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### Appendix A

#### Table A.1: Summary of calibrations I, II and III, Hardin’s void ratio function

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#### Table A.2: Summary of calibrations I, II and III, Jamiołkowski’s void ratio function

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#### Table A.3: Summary of calibrations IV and V, Hardin’s void ratio function, $m$ is a function of $f_c$

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Table A.4: Summary of calibrations IV, V. Jamiołkowski’s void ratio function, $m$ is a function of $f_c$.

<table>
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<tr>
<th>$p$ [kPa]</th>
<th>$\lambda$</th>
<th>$m_{30}$</th>
<th>$m_{40}$</th>
<th>$R^2$</th>
<th>RMSD</th>
<th>$\lambda$</th>
<th>$m_{30}$</th>
<th>$m_{40}$</th>
<th>$R^2$</th>
<th>RMSD</th>
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<tr>
<td>55</td>
<td>0.3</td>
<td>0.12</td>
<td>0.14</td>
<td>0.93</td>
<td>0.99</td>
<td>2.98</td>
<td>0.24</td>
<td>0.12</td>
<td>0.14</td>
<td>0.93</td>
</tr>
<tr>
<td>80</td>
<td>0.3</td>
<td>0.11</td>
<td>0.14</td>
<td>0.96</td>
<td>0.98</td>
<td>2.46</td>
<td>0.31</td>
<td>0.11</td>
<td>0.13</td>
<td>0.96</td>
</tr>
<tr>
<td>110</td>
<td>0.3</td>
<td>0.11</td>
<td>0.14</td>
<td>0.97</td>
<td>0.97</td>
<td>2.50</td>
<td>0.37</td>
<td>0.11</td>
<td>0.13</td>
<td>0.97</td>
</tr>
<tr>
<td>140</td>
<td>0.3</td>
<td>0.11</td>
<td>0.14</td>
<td>0.98</td>
<td>0.97</td>
<td>2.95</td>
<td>0.40</td>
<td>0.11</td>
<td>0.14</td>
<td>0.97</td>
</tr>
<tr>
<td>170</td>
<td>0.3</td>
<td>0.11</td>
<td>0.14</td>
<td>0.94</td>
<td>0.97</td>
<td>3.52</td>
<td>0.43</td>
<td>0.11</td>
<td>0.14</td>
<td>0.97</td>
</tr>
<tr>
<td>200</td>
<td>0.3</td>
<td>0.11</td>
<td>0.14</td>
<td>0.91</td>
<td>0.97</td>
<td>4.50</td>
<td>0.47</td>
<td>0.11</td>
<td>0.14</td>
<td>0.96</td>
</tr>
<tr>
<td><strong>Overall $R^2$ &amp; RMSD</strong></td>
<td><strong>0.98</strong></td>
<td><strong>3.23</strong></td>
<td><strong>0.98</strong></td>
<td><strong>3.23</strong></td>
<td><strong>0.98</strong></td>
<td><strong>2.71</strong></td>
<td><strong>0.98</strong></td>
<td><strong>2.71</strong></td>
<td><strong>0.98</strong></td>
<td><strong>2.71</strong></td>
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