Scenario-driven system identification for the specification of ground models in mechanized tunneling

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Abstract

Due to the complex interactions between the ground, the driving machine, the lining tube and the built environment, the accurate assignment of in-situ system parameters for numerical simulation in mechanized tunneling is always subject to tremendous difficulties. However, the more accurate these parameters are, the more applicable the responses gained from computations will be. In particular, if the entire length of the tunnel lining is examined, then, the appropriate selection of various kinds of ground parameters is accountable for the success of a tunnel project and, more importantly, will prevent potential casualties. In this context, methods of system identification for the adaptation of numerical simulation of ground models are presented. Hereby, both deterministic and probabilistic approaches are considered for typical scenarios representing notable variations or changes in the ground model.

Keywords: System Identification, Sensitivity Analysis, Numerical Optimization Methods, Uncertainties, Mechanized Tunneling.

1 Introduction

In July 2010 a new collaborative research center (SFB 837) started at Ruhr-Universität Bochum Germany, entitled “Interaction Models in Mechanized Tunneling”. The center consists of 14 subprojects and it is funded by the German Research Foundation (DFG). This paper is part of the work conducted in the subproject C2 “Methods of System Identification for the Adaptation of Numerical Simulation Models”.

The ground model is central to computational tunneling, where a realistic ground model is crucial for predicting the distributions and magnitudes of the strains and, consequently, reducing the surface settlements caused by the TBM propagation. Based upon the information of bore holes sunken in the target area of the tunnel alignment, the ground model describes the detailed spatial distribution of the constitutive soil properties along with the geometry of the stratification. Customary bore holes, however, provide only an approximate insight into the real world geologic realities. As a consequence, only the realization of the system identification approach can result in improved and more sophisticated numerical predictions of the spatiotemporal ground behavior induced by driving the tunnel.

For system identification, a numerical simulation model is required. This model, however, represents a complex, mechanically-hydraulically coupled and a three dimensional initial boundary value problem. Also, it is characterized by various physical nonlinearities as well as various
construction stages. For application in a later reference tunnel project, numerical simulation models are needed which enable the appropriate forward computation of typical scenarios describing relevant variations or changes in the ground model with respect to prescribed output states.

According to Figure 1, the following scenarios have been defined:

- **Scenario #1**: advance in homogeneous soil.
- **Scenario #2**: advance in two homogeneous sub-soils where the parameters and inclination of the second one are unknown.
- **Scenario #3**: advance in a homogeneous subsoil with a cubic impediment (unknown in position, dimension and material parameters) in front of the tunnel face.
- **Scenario #4**: advance in an inhomogeneous subsoil with unknown parameters and spatial distribution.

2 Methodology

Methodologically, the solution of a system identification problem is based on the concatenation of observations (measurements) and computations (numerical results) using the inverse analysis procedure. By that, the defect between the measurement-based properties and the computed ones is being minimized.

2.1 Deterministic Approach

In deterministic inverse analysis, the selected numerical model is calibrated by iteratively changing a subset of its parameters until the discrepancies between the calculated/simulated responses and the observed/measured data reach a predefined minimum (Schanz et al., 2006). This procedure is illustrated in Figure 2.

In the first step of the procedure, a priori estimation or guess of the unknown parameter set is given or a reasonable range of each parameter is defined. After that, the forward model (numerical simulation) is called for the initial guess of the parameters. Subsequently, the obtained numerical results are compared with the observed/measured data in field. The discrepancy between the two sets of data is quantified by an objective function which is to be minimized applying an optimization algorithm. The set of parameters which minimizes the objective function is the best estimated set of the unknown model parameters.

The objective function considered in this approach is the least squares criterion:
\[ f(m) = \sum_{i=1}^{N} (d_{\text{obs}}^i - g_i(m))^2 \]  \hspace{1cm} (1)

where \( g_i \) is the calculated value for the point \( i \), \( m \) is the set of the model parameters, and \( d_{\text{obs}}^i \) is the observed value at the same point.

2.2 Probabilistic Approach

Customarily, the uncertainties associated with the geotechnical applications can be categorized as follows (Baecher and Christian, 2003):

- Natural variability (aleatory/objective uncertainty) associated with the inherent randomness of natural processes, like wind flow and geologic layers.
  - In this case we take samples and draw inferences.
- Knowledge uncertainty (epistemic/subjective uncertainty) caused by the lack of data or information about events and processes, and of understanding the physical laws, where the following subcategories may be distinguished:
  - Site characterization uncertainty.
  - Model uncertainty.
  - Parameter uncertainty.

In order to include the aforementioned uncertainties (inherent in the forward model and the measured data) and the prior knowledge about the unknown parameters (indicating the trend of the parameters), a Probabilistic Approach based on Bayes’ Theorem (Thomas Bayes, 1702-1761) is incorporated in the inverse problem considered here. This approach yields a solution that provides suitable uncertainty measures (Tarantola, 2005; Lombaert et al., 2010) as follows:

- The prior information of the model parameters and the uncertainties in the observed data are represented in terms of two independent probability density functions (PDFs), \( \rho_M(m) \) and \( \rho_D(d) \) respectively, with a joint PDF:
  \[ \rho(m,d) = k \rho_M(m) \rho_D(d) \]  \hspace{1cm} (2)

where \( k \) is the normalization constant.
The effect of the modeling uncertainties is mapped by a PDF referred to as the forward model probability:

$$
\Theta(m, d) = \theta(d | m) \mu_M(m)
$$

where \( \mu_M(m) \) is the homogeneous probability density over the model space \( M \).

Combining the prior information and the forward model probability by the conjunction operation gives the probabilistic solution (see Figure 3):

$$
\sigma(m, d) = k \frac{\rho(m, d) \Theta(m, d)}{\mu(m, d)}
$$

$$
= k \frac{\rho_M(m) \rho_D(d) \theta(d | m) \mu_M(m)}{\mu_M(m) \mu_D(d)}
$$

$$
= k \frac{\rho_M(m) \rho_D(d) \theta(d | m)}{\mu_D(d)}
$$

where \( \mu_D(d) \) is the homogeneous probability density over the data space \( D \).

Once the posteriori probability in the \( D \times M \) space has been defined, the posteriori probability in the model space is given by the marginal probability density:

$$
\sigma(m) = \int_D \sigma(m, d) \, dc \, dd
$$

$$
= \int_D \frac{\rho_M(m) \rho_D(d) \theta(d | m)}{\mu_D(d)} \, dc \, dd
$$

$$
= k \rho_M(m) \int_D \frac{\rho_D(d) \theta(d | m)}{\mu_D(d)} \, dc
$$

$$
= k \rho_M(m) L(m)
$$

whereby, \( L(m) \) is the likelihood function which gives a measure of how good a model \( m \) is in explaining the data.

By assuming Gaussian PDFs for both data and model uncertainties, we have

$$
\rho_D(d) = k \exp(-0.5(d-d_{obs})^T C_D^{-1}(d-d_{obs}))
$$

$$
\theta(d | m) = k \exp(-0.5(d-g(m))^T C^{-1}_M(d-g(m)))
$$

and the likelihood function becomes Gaussian having a covariance matrix \( C_L = C_D + C_M \):

Figure 3. Conceptual depiction of the general probabilistic solution of an inverse problem.
If no prior information about the model parameters is available and the components of the observed and calculated data \((d_{\text{obs}}, g(m))\) are independent as well as identically distributed with standard deviations \(S_D\) and \(S_M\), respectively, the probabilistic solution simply becomes:

\[
\sigma(m) = k \frac{L(m)}{\exp(-0.5(d_{\text{obs}} - g(m))^T C_i^{-1} (d_{\text{obs}} - g(m)))}
\]

(7)

where \(J(m, S_D, S_M)\) is the misfit function or least squares cost function.

For fixed values of \(S_D\) and \(S_M\) over the model and the data spaces, maximizing the likelihood function equals minimizing the least squares cost function. Thus, the probabilistic approach converts to the deterministic approach.

The probabilistic solution is numerically evaluated by direct sampling. Generation of realizations of the probabilistic density solution \(\sigma(m)\) is accomplished by using the Monte Carlo Markov Chain method along with the Metropolis acceptance rules (Metropolis et al., 1953; Hastings, 1970):

Given the target probability distribution \(\sigma(m|d)\), we consider a Markov chain with a given sample \(m_n\). The next sample \(m_{n+1}\) is obtained from \(m_n\) as follows:

1. Generate a uniformly distributed sample \(U \in (0,1)\).
2. If \(U \leq \alpha\) accept \(m_{n+1} = m^*\) otherwise \(m_{n+1} = m_n\).

Repeating this sequence shows that the generated Markov Chain converges to the probability distribution function \(\sigma(m|d)\).

2.3 Sensitivity Analysis

Due to the highly nonlinear problem nature of geotechnical applications with respect to both the physical and the geometrical characteristics, the numerical simulation is normally computationally expensive. In order to make the inverse analysis efficient as well as robust, it is favorable to reduce the number of the parameters to be identified by performing a sensitivity analysis. This analysis evaluates the importance of each unknown model parameter with respect to the system response resulting in a decrease of the number of the forward calculations.

In this paper, a scaled sensitivity (SS) has been utilized. The scaled sensitivity indicates the amount of information provided by the \(i\)-th observation for estimating the \(j\)-th model parameter (Zhang et al., 2003):

\[
SS_{i,j} = \left( \frac{\partial g_j(m)}{\partial m_i} \right)_m \cdot m_j \sqrt{w_i}
\]

(9)

In Eq. (9), \(w_i\) is a weighting factor that is related to the \(i\)-th observation and based on some statistical characteristics (i.e. variance, standard deviation or coefficient of variance of the error of the
observations acc. to (Hill, 1998)). The overall model sensitivity of a given model parameter $m_j$ is computed by a composite scaled sensitivity (CSS), which is the average of the SS-values, with respect to $m_j$ (Hill, 1998):

$$CSS_j = \cosde{\sqrt{\frac{1}{N} \sum_{i=1}^{N} SS_{i,j}^2}}$$

(10)

In order to compare the CSS values for different model parameters, we use the ratio $\gamma$:

$$\gamma_j = \frac{CSS_j}{\max(CSS)}$$

(11)

As a consequence, an unknown model parameter with a small value for $\gamma$ is poorly identified using the observations involved in computing CSS.

3 Application

We use a three dimensional finite element model for scenario #1 of the tunnel excavation (Figure 1), using the FE-code PLAXIS 3D, version 2010. Since the geometry, the material properties, the initial and excavation conditions are in total symmetric with respect to a vertical plane parallel to the tunnel axis (X-axis), only one-half of the model needs to be analyzed (see Figure 4). The chosen slurry shield Tunnel Boring Machine TBM being 9 m long is simulated, along with the tunnel lining, by circular plate elements assuming linear elastic behavior. The ground is modeled by the Hardening Soil Model (Schanz et al., 1999). In Table 1, the parameters of the considered constitutive models are presented. More details about the model can be seen in (Zarev et al., 2011).

![Figure 4. The geometry and properties for a three dimensional model using Plaxis 3D.](image)

<table>
<thead>
<tr>
<th>Table 1. Material properties for the models used in tunnel simulation for scenario #1.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hardening Soil Model (soil parameters)</strong></td>
</tr>
<tr>
<td>$\varphi$ [°]</td>
</tr>
<tr>
<td>35</td>
</tr>
<tr>
<td>$m$ [-]</td>
</tr>
<tr>
<td>0.7</td>
</tr>
<tr>
<td><strong>Linear Elastic parameters</strong></td>
</tr>
<tr>
<td><strong>Tunnel lining</strong></td>
</tr>
<tr>
<td>$E$ [kN/m²]</td>
</tr>
<tr>
<td>30000000</td>
</tr>
</tbody>
</table>
In order to identify the ground model (HS-Model) parameters, the following steps have been carried out:

- The model has been run for the parameter values stated in Table 1. Hereby, the vertical displacements at the two points \( O_{12} \) and \( S_{12} \) (see Figure 4) have been saved as observation measurements \( d_{\text{obs}} \) for the whole excavation phases. Subsequently, the HS-Model parameters (i.e. \( \varphi, c, E_{\text{ref}}, E_{50}, E_{\text{ur}} \)) are considered as unknowns that need to be estimated from the data set \( d_{\text{obs}} \).
- An objective function, representing the discrepancy between the observed and calculated measurements, has been adapted based on the least squares criterion (Eq.1).
- A sensitivity analysis has been implemented according to Eqs. 9, 10 and 11 for deciding which parameters have to be identified. As a result, a parameter with a small value for \( \gamma \) (e.g. \( \gamma < 0.01 \); Hill, 1998) has a negligible effect on the considered system response; therefore, it is not identified. The analysis has shown that the five parameters of the model need to be identified (Figure 5).

![Figure 5. Sensitivity analysis results for the Hardening Soil model parameters.](image)

- The subsequent identification process has been performed following the inverse analysis procedure presented in Figure 2, where two different optimization algorithms have been applied, (i) Particle Swarm Optimization PSO (Kennedy and Eberhard, 1995), and (ii) Differential Evolution Algorithm (Storn and Price, 1995). In both cases, the objective function is minimized until the parameters match their real world values to a large extend (see Figure 6).

![Figure 6. Quality of results; forward solver = back analysis solver.](image)
In the probabilistic approach, we are currently still considering the simple case in which a homogeneous PDF for the a priori information about the model parameters is assumed. Also, Gaussian PDFs are used for both data and model uncertainties, according to Eq. 7. The covariance matrix of the measured data \( C_D \) depends usually on sensing devices used in recording the observations. In (Ledesma et al., 1995) different devices with their covariance matrices are presented that can be used in our case, as well. For the present example, an error vector with independent and identically distributed components as well as fixed variance is assumed, i.e. \( S_{D} = \text{const.} \). The model error (uncertainty) is considered to vary only over the parameter space. Along the ground domain, therefore, no uncertainties are assumed. Thus, the forward model covariance matrix (\( C_M \)) reduces to \( S_M I \) (\( S_M \) is the standard deviation, \( I \) is the identity matrix) and Eq.8 becomes:

\[
\sigma(m, S_M) = k \left( S_M^2 \right)^{-N/2} \exp \left[ -\frac{1}{2} \sum_{i=1}^{N} \left( \frac{d_{\text{obs}} - g_i(m)}{S_M} \right)^2 \right]
\]

Presently, the sampling of the posteriori probability function Eq.12 is utilizing the Monte Carlo Markov Chain method. As soon as possible, the generated realizations are to be analyzed to draw inferences about the model parameters and the standard deviation of the forward model.

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References


